JOHNSTON'S

NOTES ON ASTRONOMY,

EDITED BY

JAMES LOWE,

TRINITY COLLEGE, DUBLIN.

NOTES ON ASTRONOMY,

TOGETHER WITH A COLLECTION OF

EXAMINATION QUESTIONS.

BY

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PREFACE.

THESE "Notes" are designed to aid the student in mastering the difficulties connected with the more purely mathematical portion of the Science of Astronomy. No greater mathematical attainments are presupposed than a knowledge of Euclid, and of the elements of Algebra and Plane Trigonometry. For the physical department of the elements of Astronomy, the student may consult Brinkley's "Astronomy" (edited by Dr. Stubbs) and Dr. Ball's excellent text-books.

With respect to the style of the demonstrations given in the text, the aim is rather clearness and fulness, than mathematical elegance. The Author has had great practical experience of those points which are most perplexing to the beginner, and accordingly, has endeavoured to render assistance at such places, even at some sacrifice of conciseness.

The diagrams are numerous, an essential condition of clearness. One caution, however, should be constantly remembered; the diagrams are not drawn to scale. It would be a practical impossibility to give diagrams in which the sun, moon, and earth would appear, with any approach to accuracy of relative magnitude. Such must be the excuse for the absurdities of some of the cuts. The best preventive for any evils, likely to arise from this source, is to bear in mind the numbers expressing the actual lengths of tne lines depicted.

The test-questions at the end of the volume are all selected from examination papers.

In conclusion, the Author wishes to make acknowledgment of his indebtedness to Mr. James Lowe. To him is owing not only the appearance of these "Notes" in print, and their judicious editing, but also the greater part of whatever is practically useful in this little treatise.

S. P. J.

TABLE OF CONTENTS.

NOTES ON ASTRONOMY.

CHAPTER I.—THE SPHERE.

A SPHERE is a surface every point of which is at the same distance from a point within, called the centre. **The pole of any circle** on the sphere is that point on the sphere which is equidistant from all points of the circle.

The pole of a great* circle on the sphere is that point on the sphere which is 90° distant from every point of the circle. There are evidently two such points diametrically opposite to one another.

The angle at which two great circles intersect is equal to the length of the arc joining the poles of the two great circles.

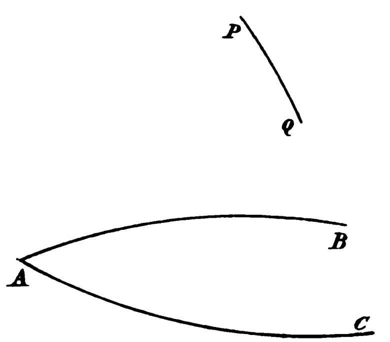

Fig 1.

Thus if P be the pole of the great circle A B, and Q the pole of the great circle A C, then the angle at A is equal to the arc P Q.

* A circle on the sphere is called great or small, according as its plane does or does not pass through the centre of the sphere.

Every great circle drawn through the pole of a great circle is perpendicular to the latter great circle.

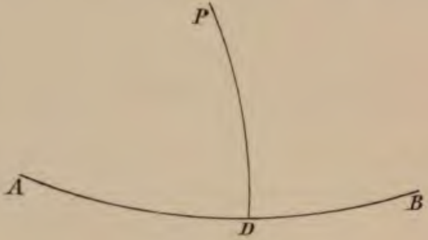

Fig II

Thus if P be the pole of A B, and if P D be any great circle through P, the angles drawn at D are right angles.

Conversely: **If any perpendicular be drawn to a great circle it will pass (when produced if necessary) through the pole of that great circle.**

Thus if at the point D in the great circle A B we draw a perpendicular great circle, it will pass through P the pole of A B.

The arcs joining the poles of two great circles will, when produced, cut both circles at right angles.

Let P be the pole of A C and Q the pole of A B. Join P Q and produce; then will the angles at C and B be 90°. For P Q is a great circle drawn through P the pole of A C, and ∴ cuts A C at right angles: and also P Q is a great circle drawn through Q the pole of A B and ∴ cuts A B at right angles.

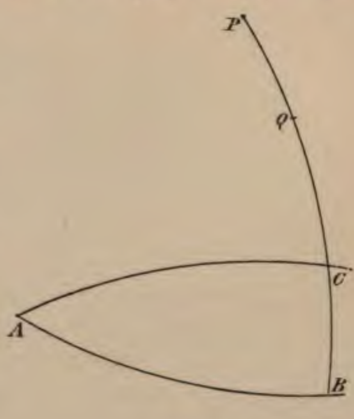

Fig III.

If a great circle be drawn to cut two great circles perpendicularly, the length of the arc intercepted on it

by the two great circles is equal to the angle at which the great circles intersect.

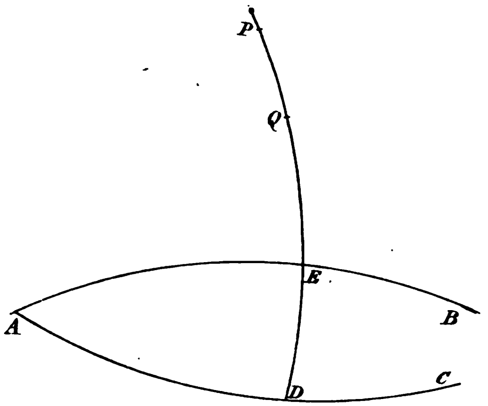

Fig IV.

If a great circle be drawn at D to cut both AC and AB at right angles in D and E, then the arc DE is equal to the angle at A.

For since DE is perpendicular to AC, when produced it passes through Q, the pole of AC. Also, for the same reason, DE produced passes through P, the pole of AB.

But since P is the pole of AB, $PE = 90°$, and since Q is the pole of AC, $QD = 90° \therefore PE = QD$: take away the common part QE, and $PQ = ED$. But $PQ =$ angle at A $\therefore ED =$ angle at A.

CHAPTER II.—DEFINITIONS.

The *celestial sphere* is a sphere, described with the eye of the spectator as centre and any radius, on which the relative positions of the stars are supposed to be indicated.

The horizon is the great circle in which the tangent plane to the earth, at any point, intersects the celestial sphere.

The *fixed stars* are those which keep always at the same relative distances: those which change their position are called *planets*.

PLANETS:

1. Mercury	5. Jupiter
2. Venus	6. Saturn
3. Earth	7. Uranus
4. Mars	8. Neptune

The *Asteroids* are small planets whose paths lie between those of Mars and Jupiter.

The four principal Asteroids are Vesta, Juno, Ceres, and Pallas.

The *diurnal motion* of the stars is their apparent motion from east to west, and arises from the rotation of the earth in the opposite direction.

The *sidereal day* is the interval which elapses before any fixed star returns to a given position.

The length of a sidereal day is 23 hours 56 minutes.

The *pole* (celestial) is that point of the celestial sphere round which all the stars seem to turn in their diurnal motion.

The *celestial pole* may also be defined as that point of the celestial sphere in which the axis of the earth intersects it.

The *zenith* is that point of the celestial sphere directly overhead.

The *nadir* is the point directly opposite the zenith.

The *zenith* is found by producing the direction of a plumb line to meet the celestial sphere.

The *celestial meridian* is the great circle (passing due north and south) through the pole and the zenith.

The *celestial equator* is the great circle, every point of which is 90° distant from the pole.

The *terrestrial meridian* is the imaginary line, at any place on the earth's surface, running due north and south.

The *terrestrial equator* is the imaginary line on the surface of the earth, circling the earth half way between the poles.

To draw a diagram of the celestial sphere :—

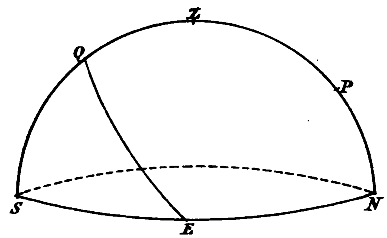

Fig V

I. Describe a semicircle S Q Z P N, to represent the *meridian.*

II. Join the ends of the semicircle by the great circle N E S, to represent the *horizon.*

III. Take the middle point, Z of the meridian S Z N, to represent the *zenith.*

IV. Take any point P (by measuring off N P equal to the latitude of the spectator), to represent the *pole.*

V. Measure off P Q = 90°, and join Q to E, the middle point of the horizon N S, by a great circle to represent the *equator.*

E.G. Draw the diagram of the celestial sphere as it would appear to a spectator in latitude 80° N. :—

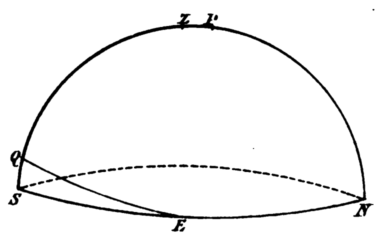

Fig VI.

The *Ecliptic* is the apparent path of the Sun among the fixed stars.

It is a great circle, and the Sun takes a year of 365·26 days to pass round it.

The *first point of Aries* is the point in which the Ecliptic intersects the equator (when the sun is passing from south to north).

The *obliquity of the Ecliptic* is the angle at which the Ecliptic cuts the equator, and is equal to 23° 28′. Symbol for the first point of Aries ♈.

The *zodiac* is a belt of stars extending 8° on both sides of the ecliptic. It is divided into twelve signs or constellations.

The *equinoxes* are the points in which the ecliptic cuts the equator.

The *solstices* are the points at which the sun is farthest from the equator.

There are two equinoxes, spring and autumn; and two solstices, summer and winter.

Spring equinox	date,	21st March
Summer solstice	„	21st June
Autumn equinox	„	23rd September
Winter solstice	„	21st December

The magnitude of a fixed star means only its degree of brightness, for with the most powerful telescopes none of the fixed stars present any visible disc. There are ten magnitudes, the brightest being the first. The lowest magnitude visible to the naked eye is the sixth.

Altitude and Azimuth.

The *altitude* of a star is the length of the perpendicular arc drawn from the star to the horizon.

The *azimuth* of a star is the intercept on the horizon between the meridian and the foot of the perpendicular, from the star, on the horizon.

To draw a diagram representing the altitude and azimuth of any star :—

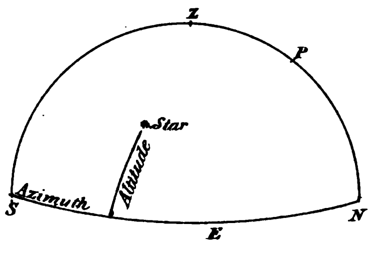

Fig VII.

Right Ascension and Declination.

The *declination* of a star is the length of the perpendicular arc drawn from the star to the equator.

The *right ascension* of a star is the intercept on the equator between the first point of Aries and the foot of the perpendicular,—on the equator,—from the star.

To draw a diagram representing the right ascension and declination of a star.

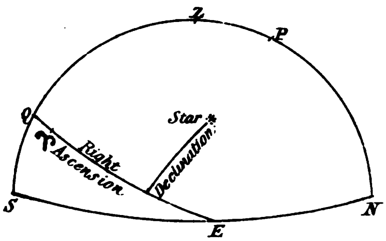

Fig VIII.

Celestial Latitude and Longitude.

The *celestial latitude* of a star is the length of the perpendicular arc drawn from the star to the ecliptic.

The *celestial longitude* of a star is the intercept on the ecliptic between the first point of Aries and the foot of the perpendicular from the star on the ecliptic.

North Polar Distance and Hour Angle.

The *North Polar Distance* of a star (N.P.D.) is the length of the arc joining the North Pole (celestial) to the star.

The *hour angle* of a star is the angle made with the meridian by the arc joining the North Pole to the star.

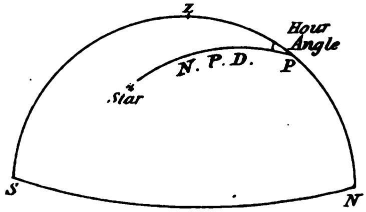

Fig IX

A *circumpolar star* is one which, at a given place on the earth, never appears to go below the horizon.

CHAPTER III.—THE FIGURE OF THE EARTH.

Proofs that the Earth is a Sphere.

I. When ships are going from us, their hulls first disappear, and their topmasts last.

II. By sailing continually to either east or west, navigators have returned to their starting place.

III. The shadow of the earth seen on the moon, when there is an eclipse, is round.

IV. Tops of mountains are seen before their bases.

V. The sun can be seen from a mountain top after it has disappeared from view on the plain.

VI. Distances on the earth, calculated on the supposition that the earth is round (a sphere), agree with the distances obtained by actual measurement.

The altitude of the pole is equal to the latitude of the place.

Let D B E A represent a section of the *earth*, C its centre E C P the axis, and therefore C P the direction of the pole.

Let A B be the equator. Let D be the place under consideration. Draw D H the tangent at D, and D Q parallel to C P. Then D Q is the direction of the pole at D, and ∴ angle H D Q = altitude of pole. Also A C D = latitude of D.

By parallels C D Q = D C E: take away the right angles C D H and A C E ∴ A C D = H P Q. Q. E. D.

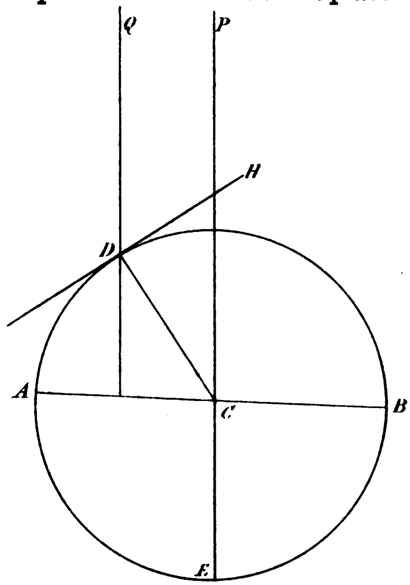

Fig X

The change in the altitude of the pole is proportional to the distance passed over north or south.

Let A D B E represent a section of the earth, and D E any two points north and south of each other; then by the preceding the altitude of the pole at E = angle A C E; also altitude of the pole at D = angle A C D; ∴ change in altitude of pole = angle A C E − angle A C D = angle D C E. But the distance D E is proportional to the angle D C E ∴ change in the altitude of pole is proportional to the distance.

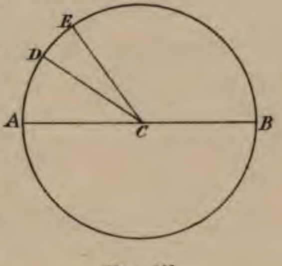

Fig XI.

To find the diameter of the earth:

(I.) Observe the altitude of the pole.

(II.) Go due north or south until the altitude has changed one degree.

(III.) Measure the distance between the two points of observation, in miles.

(IV.) Multiply by 360 and divide by $\pi = 3.14159$; the result is the diameter of the earth in miles.

To find the least declination of a circumpolar star:

Let N Z S represent the meridian, Z the zenith, P the pole, Q E A the equator.

Since the stars describe circles round the pole as centre, that circumpolar which is farthest from P will have P N for its radius; and, therefore, that star which is nearest the equator, and at the same time circumpolar, is that passing through N ∴ declination of circumpolar star with least declination = N A = A P − N P = 90° − N P = 90° − *latitude*, since N P is the latitude of the pole.

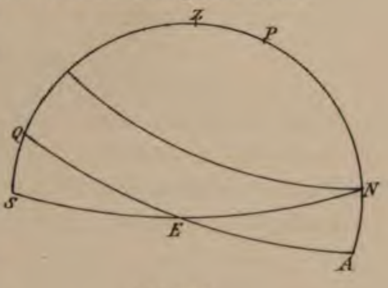

Fig XII.

Hence required declination is the *co-latitude* = Zenith distance of the pole.

Given the altitude of a star when on the meridian and knowing its declination, find the latitude of the observer.

Let A be the position of the star (the diagram is the same as usual). Then A S the altitude is known, and also A Q the declination is known $\therefore$ their difference QS is known.

But $ZS = 90°$, and $PQ = 90°$ $\therefore$ taking away the common Q Z we obtain $SQ = ZP$. But Z P is the co-latitude ($= 90° - PN = 90° -$ latitude) $\therefore$ the co-latitude is known.

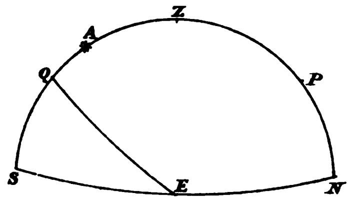

Fig XII A

The declination of a star is 34° and its altitude when on the meridian is 56° 27′. Find the latitude of the observer.

The latitude of a place is 53° 27′ and the meridian altitude of a star is 85° 26′; find the star's declination.

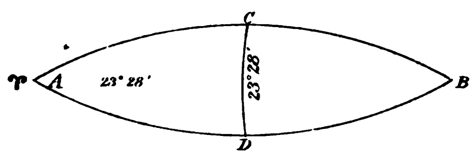

Fig XIII.

Let A C B be the ecliptic, A D B the equator; when the sun is at A (♈) its declination is 0°, when at C its declination is $CD = 23° 28'$. C being the middle point or the solstice.

Table of declinations and right ascensions of the sun:—

	RIGHT ASCENSION.	DECLINATION.
21st March	0°	0° 0′
21st June	90°	23° 28′ N.
23rd September	180°	0° 0′
21st December	270°	23° 28′ S.

Given the sun's meridian altitude on 21st June (say 46° 29′), find the latitude. We know the sun's declination on the 21st June, *i.e.*, 23° 28′: hence the problem is the same as those above.

CHAPTER IV.—REFRACTION.

Astronomical Law of Refraction.

Law: The refraction varies as the tangent of the Zenith distance.

Let the smaller circle represent a section of the earth; and the larger circle the boundary of the atmosphere. A is the centre of the earth; D the position of observer; Z the zenith; B S the direction of a ray of light from a star: B D the course taken by the same ray when refracted. Then D S′ is the *apparent* direction of the star. The angle S B S′ is the refraction; call it r. Angle S′D Z is the apparent zenith distance, call it z.

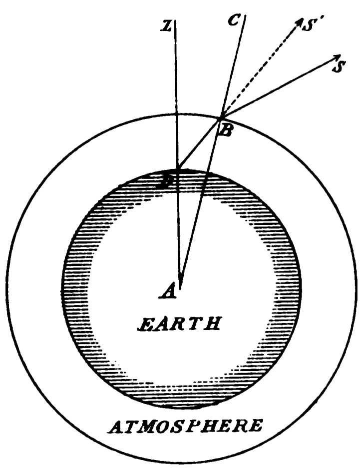

Fig XIV

Now B A and D A are very nearly parallel, since A is at such a great distance from D and B (4,000 miles). $\therefore$ angle Z D B = D B A = z.

$\therefore$ angle C B S = angle S B S′ + angle S′ B C = $r + z$.

Now from optical law of refraction.

$$\sin SBC = \mu \sin DBA \;\therefore$$

$$\sin (r + z) = \mu \sin z. \quad \text{Expand}; \;\therefore$$

$$\sin r \cos z + \cos r \sin z = \mu \sin z.$$

divide across by $\cos z$ $\therefore$

$$\sin r + \frac{\cos r . \sin z}{\cos z} = \frac{\mu \sin z.}{\cos z.}$$

$$\therefore \sin r + \cos r \tan z = \mu \tan z.$$

Now r is always a small angle, and $\therefore$ we may assume $\sin r = r$ (where r is the circular measure) and $\cos r = 1$; hence the above becomes

$$r + \tan z = \mu \tan z.$$
$$\therefore\ r = \mu \tan z - \tan z.$$
$$= (\mu - 1) \tan z = K \tan z.$$

To show that the sine of a small angle is equal to its circular measure, and its cosine equal to unity.

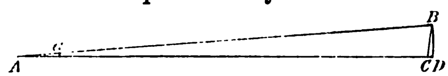

Fig XV.

Let θ be a small angle: then $\sin \theta = \frac{BC}{AB}$; but the perpendicular BC is nearly equal to BD $\therefore \sin \theta = \frac{BD}{AB}$ = circular measure: and, since AC is nearly equal to AB, $\cos \theta = \frac{AC}{AB} = 1$.

To find the constant coefficient of refraction (Brinkley's method):—

Let A B be the two points in which a circumpolar star crosses the meridian. Then by refraction the star will be raised towards the zenith, so that we shall see it apparently at A′ and B′. Let the *observed* zenith distances Z A′ and Z B′ be Z and Z′. Then the refraction A A′ is equal to K tan Z; and the refraction BB′ is equal to K tan Z′.

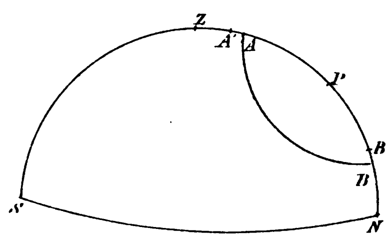

Fig XVI·

Hence true zenith distance of A is Z + K tan Z, and of B, Z′ + K tan Z.

Now, Z A + Z B = Z A + Z P + P B = Z A + Z P + P A = Z A + P A + Z P = 2.Z P, *i.e.*, Z A + Z B = 2.Z P = 2 (90° − latitude) $\therefore$ if the latitude is λ we have

$$ZA + ZB = 2\,(90^\circ - \lambda).$$

But $ZA = Z + K \tan Z$; and $ZB = Z' + K \tan Z'$.

$$\therefore\ Z + K \tan Z + Z' + K \tan Z' = 2(90^\circ - \lambda).$$
$$\therefore\ K \tan Z + K \tan Z' = 2(90^\circ - \lambda) - Z - Z'.$$
$$\therefore\ K = \frac{180^\circ - 2\lambda - Z - Z'}{\tan Z + \tan Z'},$$

Q. E. F.

To find the constant coefficient of refraction (Bradley's method) :—

Observe the apparent zenith distances of the pole-star at A and B, and let them be ζ and ζ'; then the true zenith distances Z A and Z B are respectively $\zeta + K \tan\zeta$, and $\zeta' + K \tan\zeta'$. But as on the preceding page $ZA + ZB = 2.ZP$ $\therefore$ $\zeta + K \tan\zeta + \zeta' + K \tan\zeta' = 2.ZP$ . . . . . (I).

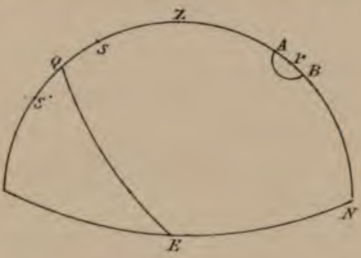

Fig XVII

Again, observe the zenith distances of the sun at summer and winter solstices, at S and S′ (when $SQ = S'Q$), and let the apparent zenith distance be Z and Z′; then the true zenith distances Z S and Z S′ are :—

$$Z + K \tan Z \text{ and } Z' + K \tan Z'; \therefore$$
$$ZS + ZS' = Z + K \tan Z + Z' + K \tan Z';$$
$$\text{also } ZS + ZS' = 2.ZQ \text{ . . . . (II)}$$
$$\text{add equations (I) and (II) } \therefore$$
$$\zeta + K \tan\zeta + \zeta' + K \tan\zeta' + Z + K \tan Z + Z' + K \tan Z'$$
$$= 2.ZP + 2.ZQ = 2(ZP + ZQ) = 2(90°) = 180°.$$

Therefore K may be found since all the other terms of the equation are known. Q. E. F.

Advantages and disadvantages of the two methods (Brinkley's and Bradley's) for finding the coefficient of refraction.

Brinkley's method requires an accurate knowledge of the latitude, but is quicker, as it only requires two observations of a circumpolar star.

Bradley's method avoids the necessity of knowing the latitude accurately, but requires six months to elapse before we can obtain the two observations of the sun.

Value of K, the constant coefficient of refraction; $\underline{K = 58''}$.

Twilight is caused by the *reflection* of light, and lasts until the sun has gone 18° below the horizon.

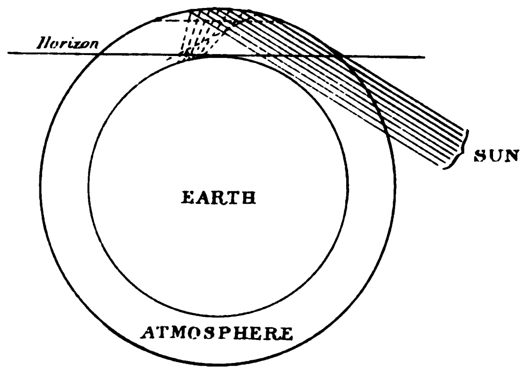

Fig XVIII.

To calculate the duration of twilight at the equator, when the sun is on the (celestial) equator.

Draw the diagram of the celestial sphere for a person on the equator. The latitude being 0°, the altitude of the pole is 0°, and therefore P is on the horizon. Draw A B parallel to the equator at a distance of 18° below, then the twilight will last while the sun is crossing the band between AB and SP at ED.

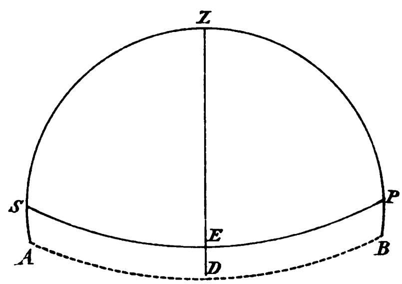

Fig XIX

But the sun takes 24 hours to pass round the complete circle of 360° ∴ to pass over E D, 18°, we have the time found by the proportion

$$360° : 18° :: 24 : \text{required time.}$$

$$\therefore \text{ required time} = \frac{18 \times 24}{360} = \frac{2 \times 24}{40}$$

$$= \frac{2 \times 3}{5} = \frac{6}{5} = 1\tfrac{1}{5} \text{ hours.}$$

∴ time of twilight = 1 hour, 12 minutes. Q. E. F.

To explain the oval appearance of the sun and moon when near the horizon. Since the zenith distance of the lower edge of the moon is greater than that of the upper edge, the refraction of the lower edge is greater than that of the upper edge, and since the lower edge is raised more than the upper edge, the two edges are brought nearer together than they really are.

Also the horizontal diameter is not altered, apparently, by refraction. Hence, while the horizontal diameter appears the same, the vertical diameter is shortened, and ∴ the disc appears flattened.

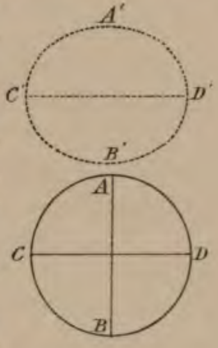

Fig XX

CHAPTER V.—PARALLAX.

Parallax is the angle subtended at a heavenly body by the radius of the earth.

Let the circle represent a section of the earth, A the position of the observer, and M a heavenly body, C the centre of the earth. Then the parallax of M is the angle A M C.

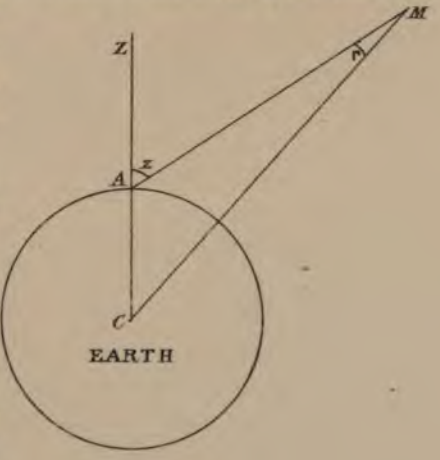

Fig XXI.

The parallax varies as the sine of the zenith distance.

In the above figure let the zenith distance of M, angle M A Z = Z ; let the parallax A M C = p. Then if the radius of earth = r and if distance CM = R, by trigonometry

Sin A M C : sin M A C :: A C : M C
but sin M A C = sin M A Z = sin Z
$\therefore$ sin p : sin Z :: r : R

$$\therefore \sin p = \frac{r \sin Z}{R}$$

but since p is always small, sin p = p

$$\therefore p = \frac{r}{R} \sin Z = K \tan Z. \qquad \text{Q. E. D.}$$

Parallax Depresses a Heavenly Body.

From A (with the same figure as the preceding) draw A M′

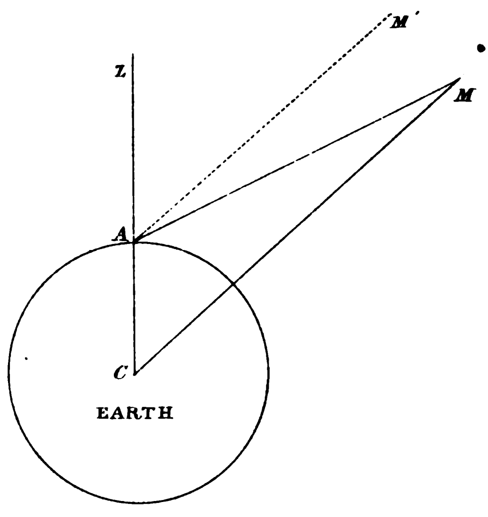

Fig XXII

parallel to C M. Then by parallels, M′A Z = M C Z. But M C Z is the zenith distance, as seen from the centre of the earth $\therefore$ M A Z the observed zenith distance exceeds by the angle M A M′ the zenith distance M′A Z, which would be observed if our observation were made at the centre of the earth.

Hence to obtain the zenith distance of a star as seen from the centre of the earth, deduct angle M A M′ from the observed zenith distance. But M A M′ = A M C (by parallels) = the parallax of M, $\therefore$ the parallax is what must be deducted from the apparent zenith

distance to give the true zenith distance, *i.e.*, that seen from the centre of the earth.

Horizontal parallax. Definition. The horizontal parallax of a star is its parallax when on the horizon.

The parallax of a heavenly body is greatest when observed on the horizon.

For the parallax varies as the sine of the zenith distance, and the sine is greatest when the angle is $90° \therefore$ the star must be $90°$ distant from the zenith, *i.e.*, it must be on the horizon.

This may also be stated thus: the horizontal parallax is the greatest.

To calculate the angle subtended at the moon by two distant places on the earth.

Let A B be the two places on the earth; M the position of the

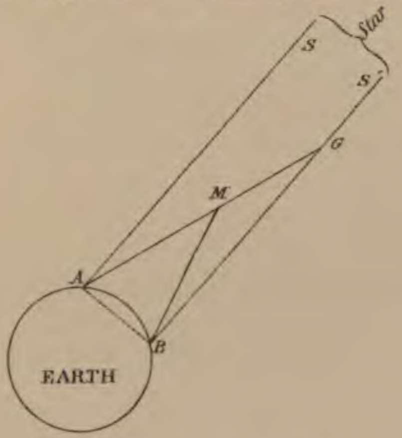

Fig XXIII.

moon and suppose a fixed star be observed in direction A S from A and in the direction B S′ from B. Then on account of the great distances of the fixed stars, A S and B S′ are practically parallel. Observe the angle S A M at A, and the angle S′B M at B. Then the angle $A M B = M G' B + M B S'$. But by parallels $M G' B = M A S$ $\therefore A M B = M A S + M B S'$ both of which are known.

Hence the angle subtended by A B at the moon is known.

To determine the moon's diameter—

By the preceding theorem we can determine in the triangle AMB, the angle at M. We can also calculate the distance A B and the angle A M B ∴ the triangle M A B can be completely determined, and the distance A M can be found. Then knowing the distance A M, and observing the angle subtended by the moon's diameter, we can calculate the diameter.

Annual parallax. Definition. The annual parallax of a heavenly body is the angle subtended at that body by the radius of the earth's orbit.

(The diurnal parallax is the angle subtended by the radius of the earth, 4,000 miles. The annual parallax is the angle subtended by the radius of the earth's orbit round the sun, 92,000,000 miles.)

When parallax is spoken of we usually mean diurnal parallax; but there is no very great confusion in not specifying which kind of parallax, when we mention the heavenly body concerned, for diurnal parallax is only used with reference to those bodies near us, and principally the moon.

Fixed stars have no diurnal parallax, because on account of their enormous distances the radius of the earth does not subtend any measurable angle, but the radius of the earth's orbit, being so very much larger than that (radius) of the earth, may subtend an appreciable though small angle.

To calculate the annual parallax of Jupiter (or any other planet).

Let E O E′ represent the earth's orbit with the sun in the

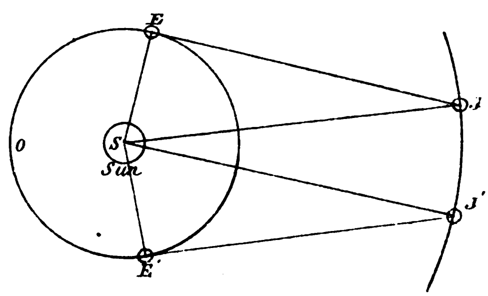

Fig. XXIV

centre, and J J′ part of the orbit of Jupiter. Observe Jupiter when the angle J E S is 90°, and note the time of the year. Again, when Jupiter has got on to J′, the earth, moving as it does faster than Jupiter, will be near E′, and observe the time at which the angle J′ E′ S is 90°. We can thus obtain the interval between the two observations. Now, the earth describes 360° round the sun in 365·26 days ∴ knowing the time between E and E′, we can obtain the angle E S E′ by a sum in proportion. Also knowing the time Jupiter takes to go round its orbit, we can in the same way obtain the angle J S J′. Hence, subtracting J S J′ from E S E′, we obtain E S J. plus E′ S J′.

But the triangles E S J and E′ S J′ are equal, because E S = E′ S, J S = J′ S, and angle E = 90° = angle E′

∴ E S J + E′ S J′ = twice E S J, hence E S J can be found, but E J S is the annual parallax.

Assuming that the annual parallax of the star α Lyræ is ·18″, calculate its distance from the earth.

Let P be the position of α Lyræ; S and E the sun and

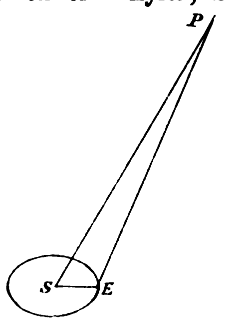

Fig XXV.

earth. Then we are granted that the angle at P is ·18″. Let x denote the distance P E, then circular measure of angle at P is $\frac{SE}{x} = \frac{92000000}{x}$.

But circular measure of ·18″ is $\frac{\cdot 18}{206265}$

$$\therefore \frac{92000000}{x} = \frac{\cdot 18}{206265} \therefore x = \frac{206265 \times 92000000}{\cdot 18}.$$

Bessel's method of determining the annual parallax of a fixed star.

If there be two stars very close together in the heavens, one bright, the other extremely faint, we may assume that the difference in brightness arises from the faint star being farther off than the bright one.

Observe at A the angular distance between the bright star P

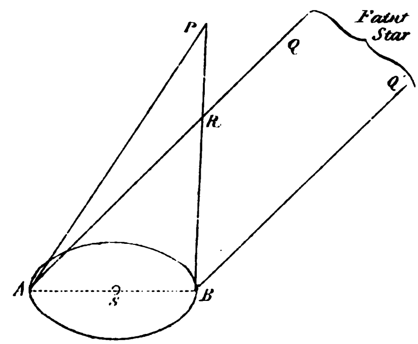

Fig XXVI.

and the faint one in the direction A Q, *i.e.* obtain the angle P A Q.

In six months time observe at B the angular distance between the same stars, *i.e.*, obtain the angle P B Q′, where B Q′ is parallel to A Q on account of the great distance of the faint star.

Now angle A P B = angle P R Q − angle P A Q, but P R Q = P B Q′ by parallels ∴ A P B = P B Q′ − P A Q and is therefore known.

But since six months have elapsed between the times of the earth being at A and B, A B is a diameter ∴ we have found the angle A P B subtended by the diameter at P, and ∴ by halving it may obtain the angle subtended by the radius, *i.e.*, the annual parallax.

To find the real diameters of the planets from their apparent diameters.

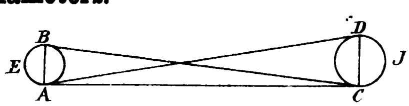

Fig XXVII

Let E be the earth and J any planet. From A let J subtend the angle D A C. Then circular measure of angle $DAC = \frac{DC}{AC}$. Similarly circular mêasure of angle $BCA = \frac{BA}{CA}$.

$$\therefore DAC : BCA :: \frac{DC}{AC} : \frac{BA}{CA}.$$

$\therefore DAC : BCA :: DC : BA$, *i.e.*,

The angle subtended by J at E : angle subtended by E at J : : diameter of J : diameter of E.

Now three terms of this proportion can be found, *i.e.*, the two angles and the diameter of E $\therefore$ the fourth term, the diameter of the other planet can be found.

CHAPTER VI.—MOTIONS OF THE SUN AND EARTH.

To Prove that the Sun Rotates.

If the sun be suitably observed, dark spots can be seen on its disc. These spots do not remain fixed, but move from east to west across the face, and are visible and invisible for equal periods. This can only arise from the sun turning on an axis.

To prove that the sun spots are on the surface of the sun.

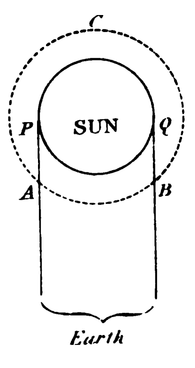

Fig. XXVIII.

If the spots be not supposed on the sun's surface, let them be small bodies moving in orbits round the sun. Let A B C represent one of these orbits. Then if lines be drawn at P Q, in the direction of the earth, the body will appear to be on the disc of the sun while passing from A to B, and it will not be seen while passing round B C A the remainder of the orbit. Hence the time the body is out of sight would be very much longer than the time it is in sight. But by observation we easily find that the sun spots are visible and invisible for equal periods $\therefore$ they cannot be off the sun's surface.

PROOFS OF THE EARTH'S ROTATION.

I. *From simplicity.* It is very much easier to suppose that the earth, which is a very insignificant heavenly body, is turning round on its own axis, than to suppose that all the visible universe must revolve once in 24 hours.

II. *From centrifugal force.* The fixed stars are at enormous distances from the earth, and if they were to revolve round the earth once a day, they would produce centrifugal forces that would be practically infinite, and the fixed stars could not be kept in their places.

III. *From falling bodies.* (First experimental proof.) If a stone be let fall from the top of a tower, it will fall slightly to the east of the foot of the tower. This is caused by the top of the tower, which is farther from the axis, moving faster than the foot of the tower.

IV. *From the Pendulum.* (Second experimental proof.)—If an observer were stationed at the north pole on the earth, and were to set a pendulum swinging in a given direction, he would, in a short time, observe the direction of its swing to veer round, and finally to turn completely round to its original direction in 24 hours. This is caused by the earth's rotation. For the pendulum has only apparently changed. It has really remained fixed in its direction, while the observer has turned round the pendulum.

The same appearances can be observed in a modified degree in lower latitudes; but not at all at the equator.

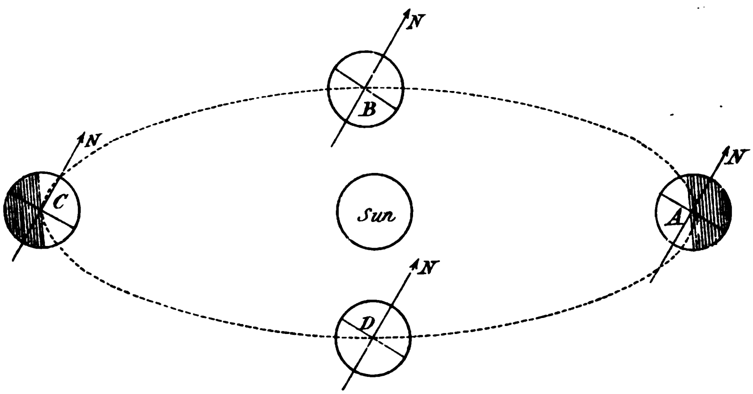

Fig XXIX

EXPLANATION OF THE SEASONS.

Let the curve represent the path of the earth round the sun, and let A, B, C, D, be four positions of the earth in its orbit at winter, spring, summer, and autumn respectively. The axis of the earth always remains parallel to itself, and ∴ always makes the same angle with the ecliptic.

At A the North Pole of the earth is turned away from the sun, and ∴ the northern half of the earth receives less light and heat from the sun than the southern half; in this position, there is winter in the north and summer in the south. *At B* both poles of the earth are at equal distances from the sun, and both the northern and southern hemispheres receive equal amounts of heat and light; it is then spring in the north and autumn in the south. *At C* the northern pole is turned towards the sun, and the southern one away, and the northern hemisphere receives more light and heat than the southern one; it is then summer in the north and winter in the south. *At D* we are in a situation similar to that at B, and both hemispheres receive equal amounts of heat and light. It is then autumn in the north and spring in the south.

CHAPTER VII.—PRECESSION AND NUTATION.

THE PRECESSION OF THE EQUINOXES.

The first point of Aries is not an absolutely fixed point, but moves slowly backward (*i.e.*, in a direction opposite to the apparent annual motion of the sun) along the ecliptic. This motion (called the precession of the equinox), takes place at the rate of 50·2″ per annum, or 1° in 72 years.

The cause of the precession of the equinoxes is the attraction of the sun and moon on the protruding matter at the earth's equator.

Since longitude is measured along the ecliptic from the first point of Aries, and since the first point of Aries is moving backward, **the longitude of each fixed star is increased by 50·2″ per annum.**

The obliquity of the ecliptic remaining a fixed quantity, the distance between the pole of the ecliptic and the pole of the equator is fixed and ∴ on account of precession, the pole of the equator appears to describe a circle round the pole of the ecliptic. It would take 26,000 years to complete the circle.

Since the pole of the equator moves among the fixed stars, the present pole star will cease to be the pole star when the pole of the equator has moved sufficiently far from that star. In 10,000 years, the star α Lyræ will be near the pole of the equator, and will be the pole star of that epoch.

NUTATION.

The obliquity of the ecliptic is not an absolutely constant quantity, but alternately increases and decreases. This change is called nutation. The total amount of the nutation is $18\frac{1}{2}''$, and its period of increase and decrease is $18\frac{2}{3}$ years.

CHAPTER VIII.—THE PLANETARY SYSTEM.

The Ptolemaic or ancient system of the universe made the earth the centre, and caused all the other heavenly bodies to revolve round the earth.

The Copernican or modern system places the sun at the centre of the planetary system and causes the planets to revolve round the sun.

The planets in order are

Inferior planets, between sun and Earth	Mercury	Interior planets, between asteroids and sun
	Venus	
	Earth	
Superior planets, orbits outside orbit of earth	Mars	
	Asteroids	
	Jupiter	Exterior planets, outside asteroids.
	Saturn	
	Uranus	
	Neptune	

Definition. The elongation of a planet from the sun, is the angle between the lines drawn from the observer to the sun and planet.

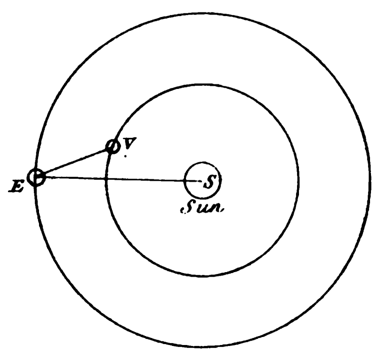

Fig XXX.

Let S be the sun, V, Venus, E, earth, then the angle V E S is the elongation of Venus from the sun.

The greatest elongation of an inferior planet is found by drawing from the earth a tangent to the inferior orbit.

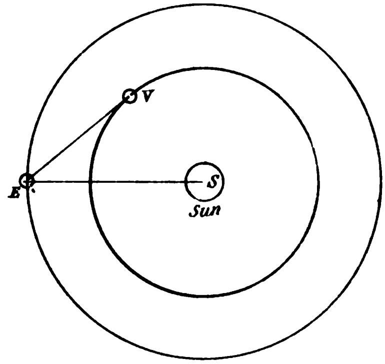

Fig XXXI

Let E be the earth at any point of its orbit. Then the relative positions of Venus with respect to Sun and earth may be found by considering the earth as stationary and Venus as moving round its orbit. Then evidently the angle V E S will be greatest when E V is a tangent to the orbit of Venus.

A planet is said to be in conjunction when it is in the same straight line with the earth and sun.

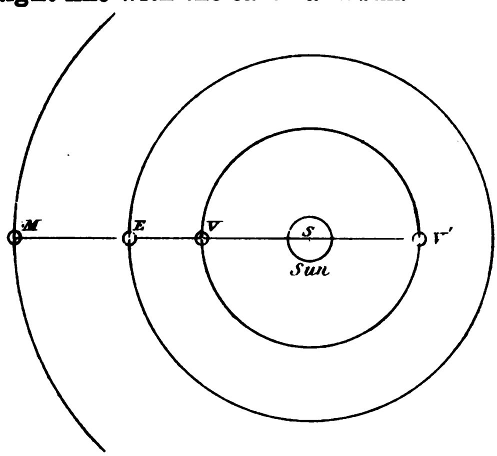

Fig XXXII.

Inferior conjunction is when the planet comes between the sun and earth. Thus in the figure Venus at V is in inferior conjunction.

Superior conjunction is when the planet is in line with the sun but on far side from earth. Thus Venus at V′ is in superior conjunction. **Opposition is when the planet is in the line joining earth and sun but on far side of earth.** Thus Mars at M is in opposition.

It is frequently necessary to use the interval between two successive conjunctions of the same kind. (This period is sometimes called the *synodic time* of the planet).

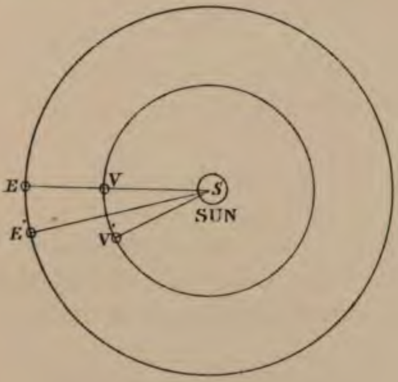

Fig XXXIII

Let E, V, S, be the positions of the earth, Venus, and sun, at an inferior conjunction. In one day let Venus get to V′ and the earth to E′. Then Venus has gained the angle V′ S E′ on the earth in one day. When this angle, V′ S E′ has increased to 360°, *i.e.*, when Venus has gained four right-angles on the earth, she will be again in the same straight line, and in inferior conjunction.

If T denote the time between two conjunctions of the same kind then $\frac{360°}{T}$ is the angle the inferior planet gains each day.

For in the time between two conjunctions, the inferior planet gains 360°. If $\therefore$ it gains 360° in T days in one day it will gain $\frac{360°}{T}$.

To find the ratio of the distances of Venus and the earth from the sun.

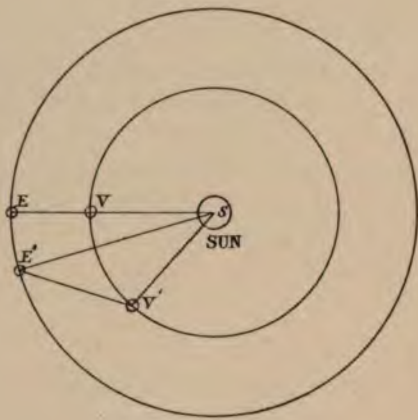

Fig XXXIV

Let E, V, S, be the positions of the earth, Venus, and sun at inferior conjunction, and at a known interval from conjunction let the earth be at E′ and Venus at V′. Then by *observation* we can obtain the angle S E′ V′. Also knowing the time from conjunction and the angle gained by Venus each day, we can find by *calculation*, the angle E′ S V′. Hence we know two angles of the triangle E′ S V′ and ∴ the third can be found. Hence the ratio of E′ S to V′ S, can be found, for it is equal to the ratio of the sines of the opposite angles, and these angles are known by above.

The periodic time of a planet is the time it takes to pass completely round its orbit.

Given the time between two inferior conjunctions of a planet, to calculate its periodic time. Let E, V, S (Fig. xxxiv.), be the positions of the earth, Venus and sun, at inferior conjunction, and after *one day* let the earth and Venus be at E′ and V′. Then if E denote the periodic time of the earth, (*i.e.*, the time the earth takes to describe 360° round the sun,) in one day the angle described by the earth is $\frac{360°}{E}$. Also if P be the periodic time of Venus $\frac{360}{P}$ is the angle described by Venus, ∴ angle E S E′ $= \frac{360}{E}$

and $VSV' = \frac{360}{P}$. $\therefore$ angle $E'SV' = \frac{360}{P} - \frac{360}{E}$. But angle gained by Venus in one day is $\frac{360}{T}$ where T is the time between two consecutive conjunctions. $\therefore \frac{360}{P} - \frac{360}{E} = \frac{360}{T}$. $\therefore \frac{1}{P} - \frac{1}{E} = \frac{1}{T}$. Now in this equation we know $E = 365\frac{1}{4}$ days, and by hypothesis we know T, $\therefore$ P can be found.

The result of preceding may be expressed in a much more general way as follows :—

Given the periodic time of one planet and the interval between its conjunctions with a second planet, to calculate the periodic time of the second planet.

By the preceding method of proof, taking the earth as the outer planet, and Venus as the inner, we have the formula.

$$\frac{1}{\left(\begin{smallmatrix}\text{Periodic time of}\\ \text{inner planet.}\end{smallmatrix}\right)} - \frac{1}{\left(\begin{smallmatrix}\text{Periodic time of}\\ \text{outer planet.}\end{smallmatrix}\right)} = \frac{1}{\left(\begin{smallmatrix}\text{Interval between two}\\ \text{conjunctions.}\end{smallmatrix}\right)}$$

Examples.—The time between two inferior conjunctions of Mercury is 116 days. Find the periodic time of Mercury.

We have $\frac{1}{P} - \frac{1}{365\frac{1}{4}} = \frac{1}{116}$ where P denotes the periodic time of Mercury; $\therefore \frac{1}{P} = \frac{1}{365\frac{1}{4}} + \frac{1}{116}$ from which P can be found.

Definition.—The *nodes* of a planet's orbit are the points in which it cuts the plane of the ecliptic. The *ascending* node is that at which the planet passes from south to north. The *descending* node is that at which the planet passes from north to south.

The brightness of a planet depends on two circumstances—(I) its distance; (II) the amount of illuminated surface visible.

The greatest width of the illuminated hemisphere of a planet, or of the moon, turned towards the earth is proportional to the external angle of elongation (at the planet), of the earth from the sun.

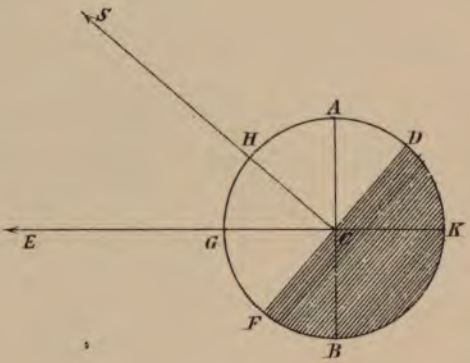

Fig XXXV.

Let C S be the direction of the sun as seen from C, the centre of the planet. Then if F D be drawn perpendicular to C S, it divides the light from the darkness on the disc of the planet. Also, if C E be the direction of the earth, the perpendicular diameter A B divides the visible portion A H G F B from the invisible portion A D B. Hence, the portion of the visible illuminated hemisphere has its greatest width equal to A H G F. Now A C K = H C F both being 90°. Add A C H to both ∴ H C K = A C F. But A C F measures the illuminated visible disc, and H C K = external angle of H C G = external angle of elongation of S from E.

Law of Gravitation. Every particle of matter in the universe attracts every other particle of matter with a force varying directly as the mass and inversely as the square of the distance.

KEPLER'S LAWS.

I. The planets move in ellipses around the sun, which is one of the foci.

II. The planets describe about the sun, equal areas in equal times.

III. The squares of the periodic times vary as the cubes of the mean distances.

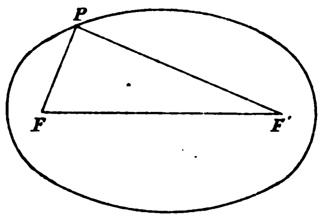

Fig XXXVI.

Law I. An ellipse is a figure described as follows: Take an endless band and place it round two fixed pegs, F and F′. Then with a pencil, at P, keeping the string stretched, run round the outline as in figure. The two points F and F′ are the *foci* of the ellipse.

If the elliptic orbit of the earth were drawn to scale no one by merely looking at it could detect any ellipticity, but would regard it as a circle.

Law II. Let P Q be two positions of a planet in its orbit, and

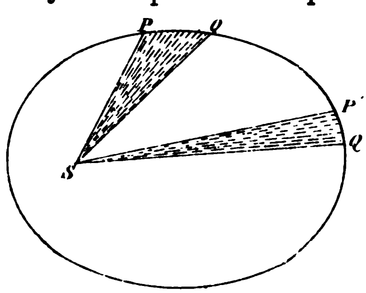

Fig XXXVII.

let S be the position of the sun. Again, let P′ Q′ be two other positions of the sun, such that the time occupied in passing from P′ to Q′ is the same as the time from P to Q. Then by Law II. the area P Q S is equal to the area P′ Q′ S.

Law III. Take any two planets, say the earth and Venus. The periodic time of the earth is 365¼ days, and let the periodic time

of Venus be V. The mean distance (or average distance) of the earth is 92,000,000 miles, and let R be the mean distance of Venus, then by Law III.

(the square of periodic time of earth,) is to
(the square of periodic time of Venus) as
(the cube of the mean distance of the earth), is to
(the cube of the mean distance of Venus),

or in numbers,

$$(365\tfrac{1}{4})^2 : V^2 :: (92,000,000)^3 : R^3.$$

Bode's Law.

To obtain the relative distances of the planets from the sun.

I. Write down the series of numbers.

0	1	2	4	8	16	32	64	128

II. Multiply each term by 3.

0	3	6	12	24	48	96	192	384

III. Add 4 to each term obtaining finally

4	7	10	16	28	52	100	196	388

Then the last series of numbers represents the distances of the planets in order from the sun, the earth being represented by 10, Thus:

4	Mercury.
7	Venus.
10	Earth.
16	Mars.
28	Average distance of asteroids.
52	Jupiter.
100	Saturn.
196	Uranus.
388	Neptune.

Assuming Bode's Law and Kepler's Third Law, calculate the periodic time of Saturn.

By Bode's Law the mean distance of the earth is 10, and that of Saturn is 100. $\therefore$ by Kepler's Third Law

(Earth's periodic time)^2 : (Saturn's periodic time)^2 : : $(10)^3 : (100)^3$.

But earth's periodic time we may take as 1, *i.e.*, 1 year.

$$\therefore\ 1 : (\text{Saturn's periodic time})^2 :: 1000 : 1000000;$$

$$\therefore (\text{Saturn's periodic time})^2 = \frac{1000000}{1000}$$

$$= 100;$$

$$\therefore\ \text{Saturn's periodic time (in years)} = \sqrt{1000}.$$

To prove that the velocities of two planets are inversely as the square roots of the radii of their orbits.

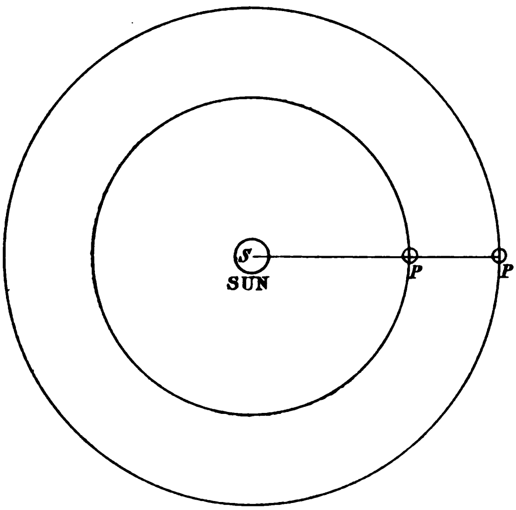

Fig XXXVIII.

Let P, P' be any two planets. T, T' their periodic times. r, r the radii of their orbits. v, v' their velocities in their orbits.

Now by Kepler's Third Law $T^2 : T'^2 :: r^3 : r'^3$. But T the periodic time of P is equal to the space divided by the velocity, and the space is the circumference of its orbit $= 2\pi r$ $\therefore$ $T = \frac{2\pi r}{v}$

and in a similar manner $T' = \frac{2\pi r'}{v'}$. Substitute these values in the proportion above. $\therefore$

$$\frac{4\pi^2 r^2}{v^2} : \frac{4\pi^2 r'^2}{v'^2} :: r^3 : r'^3$$

Multiply the extremes and means. $\therefore$

$$\frac{4\pi^2 r^2 r'^3}{v^2} = \frac{4\pi^2 r'^2 r^3}{v'^2}$$

Divide both sides by $4\pi^2 r^2 r'^2$ $\therefore$

$\frac{r'}{v^2} = \frac{r}{v'^2} \therefore \frac{\sqrt{r'}}{v} = \frac{\sqrt{r}}{v'}$ by taking square root of both sides.

$$\therefore v : v' :: \sqrt{r'} : \sqrt{r}$$

CHAPTER IX.—THE MOON AND COMETS.

The moon's periodic time round the earth is 27 days 7 hours.

The moon's orbit is inclined at an angle of 5° to the ecliptic.

The moon's nodes are the points in which its orbit intersects the plane of the ecliptic.

The moon's nodes are not fixed, but move backward along the ecliptic at the rate of 19° per annum. Thus the nodes pass completely round the ecliptic in $18\frac{2}{3}$ years, the same time as the period of the nutation.

The distance of the moon is 60 times the earth's radius, or 240,000 miles.

THE MOON'S PHASES.

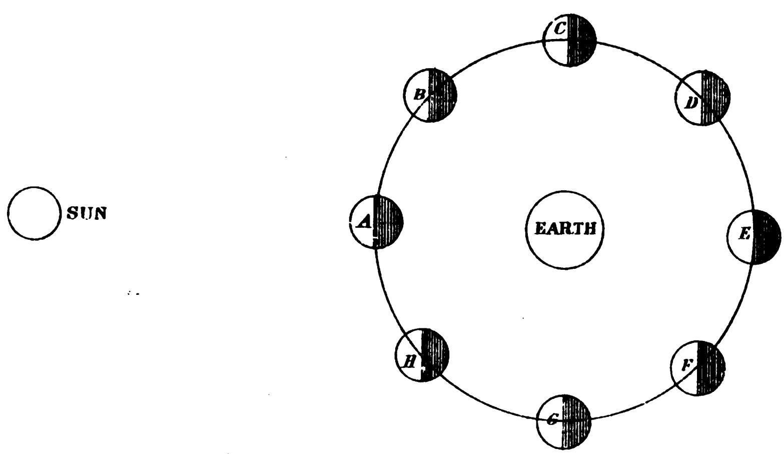

Fig XXXIX.

The moon in passing round her orbit has only that hemisphere which is turned towards the sun, lit up by the sunlight. Let A B C, &c., be consecutive positions of the moon. Then at A the light hemisphere is turned completely away from the earth, and we see only the dark side, or, there is no moon. At B we can see a small portion of the illuminated portion and the moon will be a thin crescent. At C the moon will appear very nearly equally divided by light and darkness. At D the light portion predominates. In this position the moon is said to be *gibbous*. At E we see the whole disc illuminated, or, full moon.

From E onwards, the moon's phases recur as in the first half, except that they are in reverse order.

Earth Light.—When the moon is a thin crescent, the dark portion of the disc is sometimes seen lit up by a very pale light This is caused by the light from the earth; the sunlight falling on the earth being reflected from the earth in the same way as it is reflected from the moon.

The moon is said to be dichotomised when it is exactly halved by the line dividing light from darkness.

Ancient method of finding the distance of the sun from the earth by observing the moon when dichotomised.

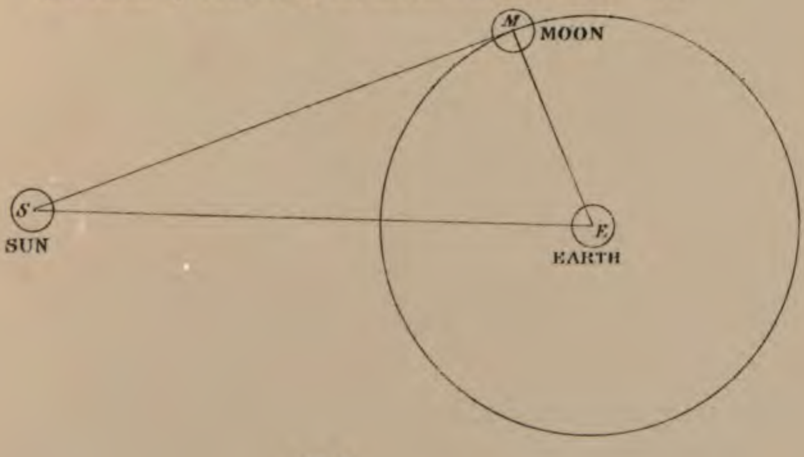

Fig XI.

If from the sun we draw S M a tangent to the moon's orbit, the moon will appear dichotomised when at M, for then M E is at right angles to S M and $\therefore$ M E separates the light from the darkness. $\therefore$ one half of the moon will appear bright, the other half dark. At this instant *i.e.* when the moon is exactly halved, observe the angle S E M (the moon's elongation) and let it be E. Then since the angle at M is a right angle $\frac{SE}{EM} = \sec. E$

$\therefore SE = EM \sec. E$. But $EM = 60 \times$ earth's radius $\therefore$ S E can be found.

Objection to this method. It is almost impossible to observe the precise moment at which the moon's disc is halved.

The Moon's Periodic Time.

Definition. A *lunation* is the time between two successive conjunctions or oppositions of the moon with sun. This is also called a synodic month. A lunation is $29\frac{1}{2}$ days.

To find the Periodic Time of Moon.

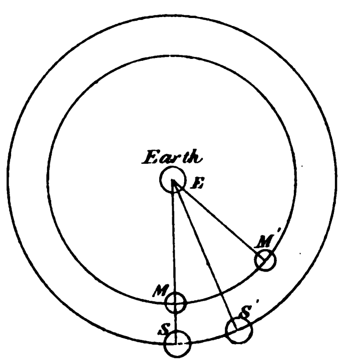

Fig XLI.

Let E, M, S, be the positions of the earth, moon, and sun at a conjunction, and assume the earth to remain fixed and the sun to move Let M denote the moon's periodic time; S, the sun's

periodic time, and L, a lunation. Let M′ S′ be the positions of the moon and sun on the next day. Then the moon has gained M E S′ on the sun, but it gains 360° in L days $\therefore$ in one day it gains $\frac{360^\circ}{L}$ $\therefore$ $M'ES' = \frac{360}{L}$. But the angle passed over by the moon in one day is $\frac{360}{M}$ and $\therefore$ $M'EM = \frac{360}{M}$, similarly $S'ES = \frac{360}{S}$ $\therefore$ since $M'ES' = M'EM - S'ES$ we have $\frac{360}{L} = \frac{360}{M} - \frac{360}{S} \therefore \frac{1}{L} = \frac{1}{M} - \frac{1}{S}$, now $S = 365\frac{1}{4}$ and $L = 29\frac{1}{2}$ $\therefore$ M can be found.

THE MOON'S LIBRATIONS.

The moon turns round its axis at such a rate that it turns once completely round its axis in every circuit it makes of the earth. Hence it keeps the same face constantly turned towards the earth.

But on account of inequalities in the rates of motion, we can sometimes see some small portion of that hemisphere which is turned from us. These inequalities produce thus what are called the moon's *librations*. There are three librations.

I. *Libration in longitude.* When the moon's velocity round the earth, (which is variable) is faster than the moon's velocity of rotation, part of the distant hemisphere will be brought into view before the rotation round the moon's axis has carried it back again.

II. The *Librations in latitude.* The moon's axis is not exactly perpendicular to the plane of its orbit, and $\therefore$ in some positions there is more visible at the northern extremity of the axis, than is usually the case; and similarly at the south pole.

Fig XLII.

Thus let M, M′, be two positions of the moon on opposite sides of the earth. When at M we can see that part between C and A, in addition to that portion which we usually see bounded by A B. Also at M′ we see the additional portion B′ D′.

III. *Diurnal libration.* At its rising we are looking at the moon from a point of view very different from the point of view of the moon when it is setting. Thus at rising we see slightly round the western edge, and at setting slightly round the eastern edge.

HARVEST MOON. About the time of the autumnal equinox if there be a full moon it will be observed to rise for several nights at about the same time by the clock, instead of being, as is usual, 40 minutes later each day. This phenomenon is called harvest moon.

The moon has no atmosphere.

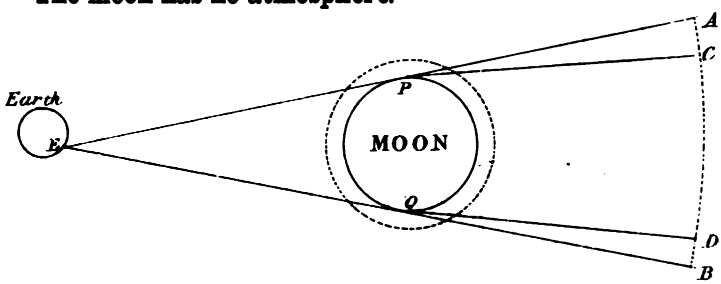

Fig XLIII

Let the moon be supposed fixed and assume the stars to move past it instead of the moon past them. Drawing tangents, (from E the position of an observer on the earth), to the moon, a star is hidden while passing from A to B. But if there were a lunar atmosphere the star's light would be bent by refraction and would reach the earth by some such path as C P E. Hence the star would not disappear until it had reached C and would for the same reason reappear at D before it had reached E. Therefore the total time of obscuration would be diminished if there were an atmosphere; but there is no perceptible diminution ∴ there can be no atmosphere.

Relation between Comets and Meteors.

Comets are luminous bodies moving in orbits which extend very far beyond the planetary system. Meteors or shooting stars are small masses moving in space and probably following in the track of the comets. They are not visible until they dash with great velocity into the earth's atmosphere. There the great friction with the air produces such an amount of heat that they become luminous and are reduced to vapour. There are two remarkable "showers" of meteors each year, early in August and about the middle of November. This is probably caused by the earth dashing through the stream of small masses following in the wake of Tempel's comet.

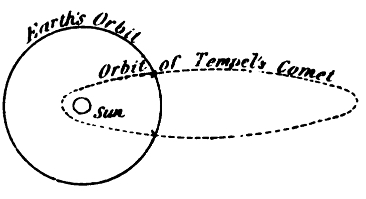

Fig XLIV.

Let the dotted line represent the orbit of Tempel's comet. Then it is very probable that throughout this orbit there are numerous small bodies continually moving round. The earth's orbit intersects this twice and then those bodies which happen to come into the region of the earth's atmosphere produce the meteoric showers then visible.

CHAPTER X.—ECLIPSES.

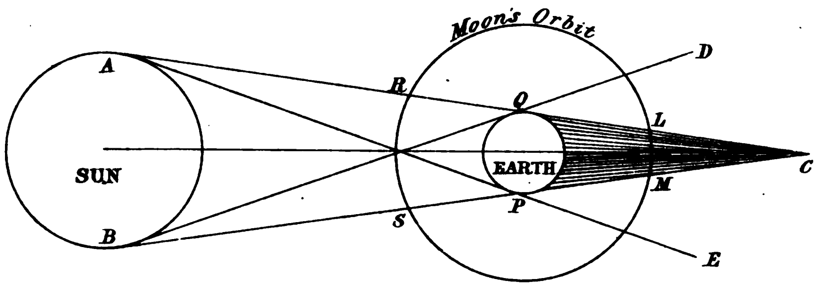

Fig XLV.

Let the tangent lines A C and B C be drawn from the sun to the earth. Then it is evident that the region C P Q is completely shut out from the light of the sun. This is called the *umbra.*

Let the tangent lines A E and B D be drawn. Then in the region C Q D part only of the sun is obscured by the earth. Similarly for the region E P C. This is called the *penumbra.*

Whenever the moon in her orbit passes through the shadow, or umbra, there is an eclipse of the moon (*i.e.*, when the moon is in L M). When the moon passes between R and S, its shadow will fall on the earth, and there will be an eclipse of the sun.

Eclipses of the sun occur more frequently than eclipses of the moon, but are less frequently visible at a given place. To explain this: Evidently from the preceding figure, R S is greater than L M, ∴ the moon will be more frequently in R S than in L M hence we may expect more eclipses of the sun than of the moon. But when an eclipse of the moon cccurs, it is visible over a whole hemisphere; for the eclipse can be seen wherever the moon is visible, that is at any place on the hemisphere turned towards the moon. On the other hand, when the sun is eclipsed, the phenomenon is visible at but few places on the earth; namely those

parts of the earth on which the moon's shadow falls, and it is not likely that any particular place will be within the region of the moon's shadow.

To find the vertical angle of the earth's shadow.

Fig XLVI.

From the sun draw tangents A C, B C to the earth. Then the angle at C is the angle of the earth's shadow. From E the centre of the earth draw E A tangent to the sun, and join the centres S and E of the sun and earth : S and E are evidently in the same straight line with C. Then angle P C E = half of angle of earth's shadow; but P C E = A E S − C A E by Euclid 32 of Book I. Now A E S is the apparent angular radius of the sun's disc, and P A E is the angle subtended at the sun by the earth's radius P E when the sun is in the horizon of P ∴ angle P A E = sun's horizontal parallax ∴ half vertical angle of earth's shadow = sun's apparent radius − sun's horizontal parallax.

To find the height of the earth's shadow.

By the preceding we can calculate the angle P C E. But $\frac{EC}{EP} = \text{cosec. } PCE$ ∴ E C = E P cosec. P C E ∴ E C or the height of the earth's shadow is known.

Practically we may for *this* purpose assume P C E = A E S, the sun's apparent radius, for A E and A P are very nearly parallel on account of the great distance of the sun.

To calculate the angular radius of the earth's shadow at the distance of the moon.

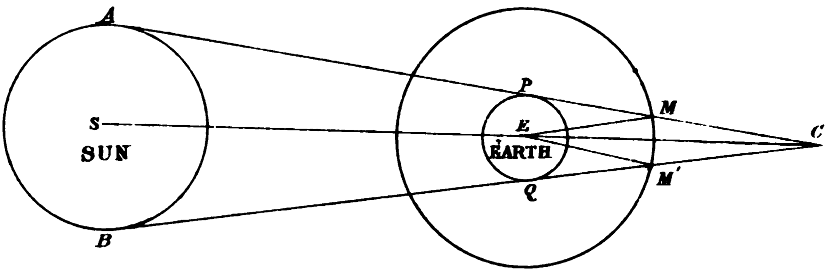

Fig XLVII

Let the larger circle with centre E represent the moon's orbit. Then if a cross section of the earth's shadow were taken at M M' it would be a circle, M M' would be its diameter, and angle M E C its angular radius.

Now M E C = E M P − E C P

= E M P − ½ vertical angle of cone.

But E M P = angle subtended at moon by radius E P of earth, when the moon is in the horizon of P ∴ E M P = moon's horizontal parallax.

∴ M E C (angular radius of shadow)

= moon's horizontal parallax − ½ vertical angle of cone.

To explain why an eclipse of the moon does not occur at every opposition.

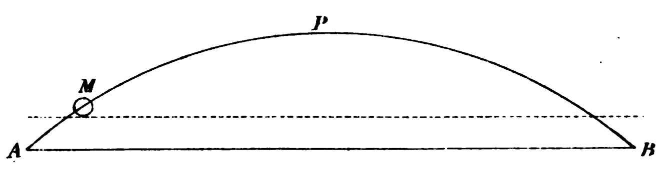

Fig XLVIII

Let A M P B represent the moon's orbit.

Let A B represent the ecliptic, then since the earth moves along the ecliptic, the earth's shadow will also move along the ecliptic, and let the dotted lines be the limits of the earth's

shadow on either side of the ecliptic. Now if the moon be near A, (one of the nodes of her orbit), when in opposition, there will be an eclipse, but if the opposition occur when the moon is at M, the shadow will be below the moon, and there will be no eclipse. In general if the moon be at too great a distance from the node, it will be above the shadow and there can be no eclipse.

Definition. The ecliptic limits are the distances from the node at which an eclipse can or must occur.

The major ecliptic limit is the greatest distance of the moon from the node at opposition at which an eclipse can occur.

The minor ecliptic limit is the greatest distance of the moon from the node at which an eclipse must occur.

To find the ecliptic limits.

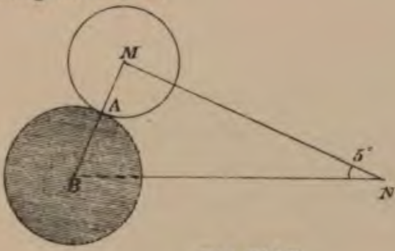

Fig XLIX.

Let B be the centre of the shadow and M the centre of the moon, B N the ecliptic and M N the moon's path. Then when the two circles just touch we have the greatest distance of M from the node in order that an eclipse can occur, provided B M be perpendicular to M N, so as to obtain the least distance of B from the moon. Now in triangle B M N, angle at N = 5°, angle at M = 90° and B M = moon's radius + shadow's radius ∴ M N can be found. But by above this is the ecliptic limit.

CHAPTER XI.—THE DISTANCE OF THE SUN.

The importance, from an astronomical point of view, of the transits of Venus, arises from the fact that observations of the transit afford us the most accurate measurements of the sun's parallax and therefore of the sun's distance.

There are two principal methods of obtaining the sun's parallax from the transit of Venus, Delisle's and Halley's.

DELISLE'S METHOD.

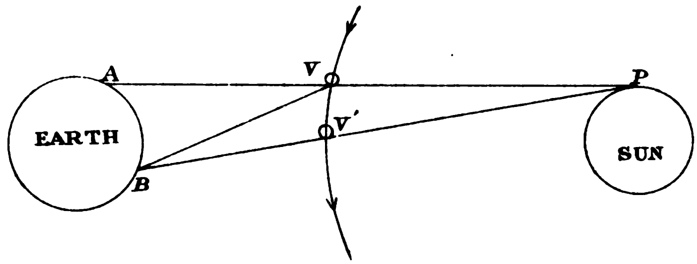

Fig L.

Let A and B be two stations on the earth, then drawing tangents A P, B P from A and B to the sun, first contact of Venus with the sun's disc will be seen from A when Venus is at V and and from B when Venus is at V′. Now we know the number of seconds of arc that Venus passes through per hour, and we can find the interval of time between Venus being at V and at V′ ∴ we can find the angle V B V′ or the angle described round B by Venus in the lapse between first contact at A and first contact at B. But sine V P B : sine V B P : : V B : V P. Now the ratio of the distances of Venus from earth and sun is a known ratio and angle V B P is known ∴ V P B can be found, ∴ we know the angle subtended by A B at sun and can ∴ find angle subtended by earth's radius at sun, *i.e.*, obtain the parallax.

HALLEY'S METHOD.

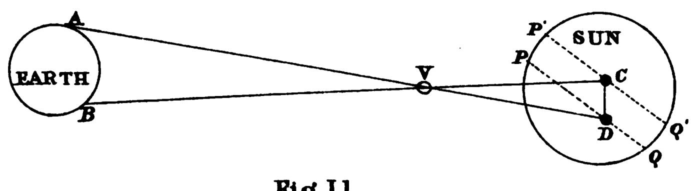

Fig Ll.

Let A, B, be two stations on the earth and let Venus be passing between the sun and earth at V in an upward direction, perpendicular to the plane of the paper. Then the observer at A will see Venus as a black spot on the sun's disc in the line A V at D. Also this black spot will cross the sun's disc in a straight line P Q. Similarly an observer at B will observe Venus moving along a parallel straight line P′ Q′. Now the triangles A V B and C V D are practically isosceles triangles, and they have the angles at V equal, therefore they are similar and ∴ C D : AB : : V D : A V. But the ratio of V D : A V, *i.e.*, the ratio of the distances of Venus from sun and earth, is known, and A B is known in miles, ∴ C D the fourth term of the proportion can be found. Now when we know the relative positions of two parallel chords of a circle, and the actual distance between them, we can calculate the diameter of the circle. Applying this principle to this case we can obtain the sun's diameter. Further we know by observation the angle the sun's diameter subtends, and ∴ by trigonometry can find the distance.

To find the sun's distance by observations of Mars when in opposition.

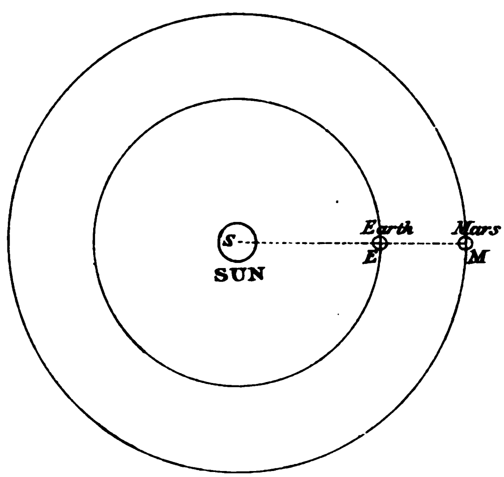

Fig LII.

Let S be the sun, E the earth and M Mars in opposition. Now by Bode's law the distances E S, and M S are 10 and 16 respectively ∴ the distance E M is 6 or little more than half the distance E S. Hence we are more likely to obtain the distance of Mars correctly by direct observation, than if we tried to measure E S directly. And if we can find E M we can by the proportion S E : E M : : 10 : 6 find S E.

An obsolete method of calculating the distance of the sun from observations of the moon has been described in Chapter IX.

CHAPTER XII.—INSTRUMENTS.

The principal instruments used in observatories are the Mural Circle (rather out of date), the Transit Instrument, the Equatorial, and the Altitude-azimuth Instrument.

The Mural Circle is used principally for finding the altitudes or zenith distances of stars, and hence determining their declinations.

The Transit Instrument is employed for finding the right ascensions and declinations of stars.

The Equatorial is used for observations of stars when off the meridian.

The Altitude-azimuth is also used for observations off the meridian. The altitude-azimuth may be concisely described as a transit-instrument mounted on a horizontal turn-table.

To obtain the declination of a star from observations of its zenith distance taken by means of either the mural circle or the transit.

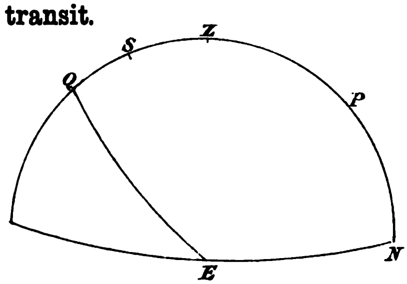

Fig LIII.

Let the semi-circle represent the meridian. Measure off N P = latitude of observer, then P is the pole. Let Z be the zenith. Make P Q = 90° then Q is the point in which the equator cuts the meridian. Let a star be observed when crossing the meridian at S and let its zenith distance Z S be observed.

Now P Q = 90° and Z N = 90° ∴ P Q = Z N take away Z P and we get Z Q = P N = latitude. Now by observation S Z is known, hence subtracting S Z from Q Z we obtain S Q, the required declination.

Ex. The observed meridian zenith distance of a star is found to be 23° 15′ 20″. Calculate the star's declination, having given the latitude of the observer 53° 25′ 37″.

To obtain the right ascension of a star from observations taken with the transit.

Note, by the astronomical clock, the time when the first point of Aries crosses the meridian. Note also the time at which the given star crosses the meridian. Then the interval in time between the two transits, turned into degrees at the rate of 15° for each hour gives the right ascension of the star.

To show that by taking readings at opposite ends of a diameter of a graduated circle, we avoid errors in centering the circle.

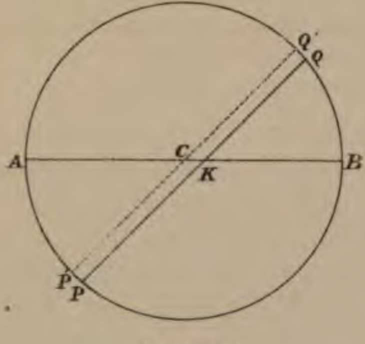

Fig LIV

Let P Q be a transit telescope pointed towards a star whose altitude we wish to find. And let the axis of the telescope be at K not coinciding with the centre of the circle on the rim of which we read off the degrees. Draw P′ Q′ through C parallel to P Q. Then if the instrument were correctly adjusted, we should read off B Q′ or A P′ as the required altitude. But what we actually read off is B Q and A P. Now

$$BQ = BQ' - QQ'$$

$$\text{and } AP = AP' + PP'$$

$$\therefore AP + BQ = BQ' + AP' \text{ (since } PP' = QQ'\text{)}, = 2\,BQ'$$

Hence dividing across by 2, $\frac{AP + BQ}{2} = BQ'$ or, in words, half the sum of the actual readings gives the true reading.

Definition.—The line of collimation of a telescope is the line joining the centre of the object glass to the centre of the eye glass.

There are three necessary adjustments of a transit instrument, and their corresponding errors when not accurately adjusted.

I. The level adjustment. The axis on which the telescope turns must be exactly horizontal.

II. The collimation adjustment. The line of collimation must be exactly perpendicular to the axis on which the telescope turns.

III. The deviation adjustment. The axis of the telescope must be placed so that it will not deviate from a direction of due east and west.

To determine whether there be any deviation error.

Observe a circumpolar star at its upper crossing of the meridian, and note the interval that elapses before the star crosses the meridian below the pole. Note also the interval that elapses before the next crossing above the pole. Compare these two intervals and only if they are exactly equal will there be no deviation error.

HADLEY'S SEXTANT.

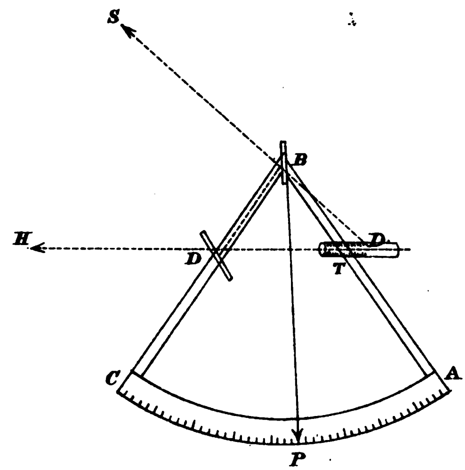

Fig LV

The instrument can be briefly described as follows. Its body is a triangular metal frame, A, B, C, the curved base of which is graduated. At B there is a small mirror fastened to the radius, B P, the radius being moveable round a pivot at B. At D there is a second mirror partly unsilvered so that it can be seen through. It is fastened rigidly to the radius C B, so that it is parallel to A B. Finally a small telescope is fastened to A B, directly opposite D.

To use the sextant in finding the altitude of a star, hold the sextant in a vertical plane and in such a position, that on looking through the telescope the sea horizon is seen through the unsilvered portion of D. Move the pointer B P (thereby turning the mirror at B), until the star is seen reflected from D, and continue to move the pointer until the star appears to be just on a level with the apparent horizon. Then the reading A P when doubled gives the required altitude.

Principle of Hadley's Sextant.—If a ray of light be

reflected by two mirrors, the angle between the original ray and the final direction, is double the angle between the mirrors.

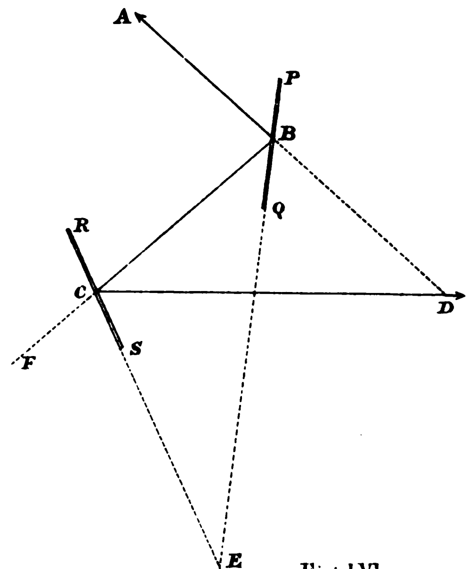

Fig LVI.

Let P Q, R S, be the mirrors, A B C D the course of the ray of light. Then producing A B to D, angle A B D is the angle between the original and final direction of the ray; and producing P Q and R S, the angle at E is the angle between the mirrors. Now, since the angles of incidence and reflexion are always equal, A B P = C B Q. But A B P = D B E $\therefore$ C B E = D B E. Similarly B C R = D C E, but B C R = E C F $\therefore$ E C F = D C E.

Since C B E = D B E, C B E = $\frac{1}{2}$ C B D

and since E C F = D C E, E C F = $\frac{1}{2}$ D C F

$\therefore$ subtracting E C F − C B E = $\frac{1}{2}$ (D C F − C B D)

but by Euc. 32 of Book I. E C F − C B E = angle at E

and D C F − C B D = angle at D

$\therefore$ angle at E = $\frac{1}{2}$ angle at D. Q. E. D.

CHAPTER XIII.—ABERRATION.

To explain the way in which the velocity of light was discovered.

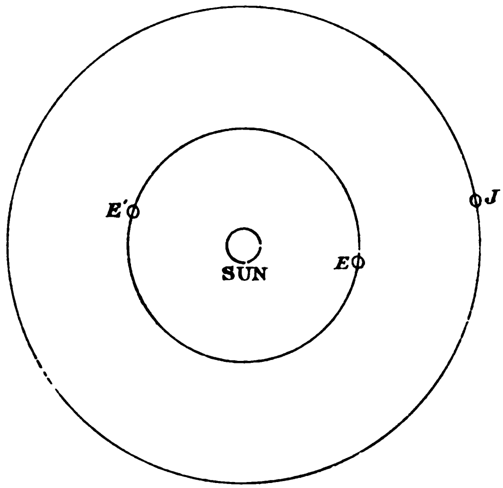

Fig LVII

The times of the eclipses of the satellites of Jupiter were calculated from observations taken when the earth was at a point of its orbit near Jupiter. In six months when the earth was at E′ the calculated times were compared with the actually observed times and were found to differ by about quarter of an hour, the observed times being later than the calculated times. This could only be explained by assuming that the extra time was that occupied by the light in passing over the increased distance. The *distance* was calculated and the *time* occupied was known, hence the velocity was obtained.

Velocity of light thus calculated is about 192,000 miles per second.

Definition. **The aberration of a star is an apparent**

displacement the star receives owing to the combined effect of the velocity of the earth and the velocity of light.

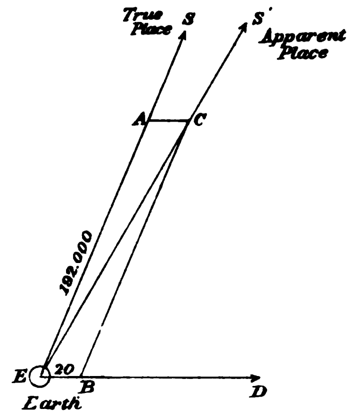

Fig LVIII

Let E be the earth moving in the direction E D, and let a star be in the direction E S. Measure off along E S a distance E A proportional to the velocity of light, and along E D, a length E B proportional to the velocity of the earth in its orbit. Then combining these two velocities, the light will *appear* to come along the diagonal of the parallelogram E C, and ∴ the star will appear to be in the direction E S′.

Hence the angle S′ E S is the angular displacement of the star S and is ∴ the aberration of that star.

To calculate the aberration.

With the same figure as before, from the parallels A E, C B, the aberration AEC = angle ECB and sin ECB : sin CEB : : EB : BC.

But by construction E B : B C : : velocity of earth : velocity of light ∴ sin E C B : sin C E B : : vel. of earth : vel. of light

$$\therefore \sin ECB = \frac{\text{vel. of earth} \times \sin CEB.}{\text{vel. of light.}}$$

But E C B is always a very small angle ∴ sin E C B = E C B, ∴

$$\text{aberration } ECB = \frac{\text{vel. of earth}}{\text{vel. of light}} \times \sin CEB$$

Now the angle C E B (*i.e.* the angle between the direction of the earth's motion, and the apparent direction of the star) is called the "*Earth's way*" $\therefore$ finally aberration = K sin (earth's way), where $K = \frac{\text{vel. of earth}}{\text{vel. of light.}}$

The actual value of K is 20."45 nearly.

Effect of aberration on the apparent positions of the stars.

I. If the star is in the pole of the ecliptic it appears to describe in the course of a year, a small circle about its true position.

II. If the star is in the ecliptic it appears to move forward and backward along a very short length of the ecliptic.

III. If the star be anywhere between the ecliptic and its pole it moves in a small ellipse round its true position.

To find that point in the heavens towards which the stars aberrate.

The required point is by the preceding proof of the formula for aberration, that point of the heavens towards which the earth is going. This latter point is the point in the ecliptic which is 90° behind the sun, for at any instant the earth is moving in the direction of the tangent to its orbit; but the tangent is in the ecliptic, and makes an angle of 90° with the radius drawn from the earth to the sun.

CHAPTER XIV.—TIME.

For two reasons there are inequalities in the time as indicated by the sun—firstly, the earth's unequal motion in its orbit, sometimes fast, sometimes slow; and secondly, the apparent motion of the sun along the ecliptic instead of along the equator. In order to obtain a regular measurement of time, astronomers had recourse to a fictitious sun which is supposed to move along the equator at such a regular rate that it completes the circuit in the same time that the true sun gets round the ecliptic. To this sun the name "Mean" sun was given, and the corresponding time is called, "Mean Solar Time."

Definition: Mean time is that which would be indicated by a sun moving in the equator at the average rate of the true sun in the ecliptic. Clocks indicate mean time.

Definition: The equation of time is the difference between mean solar time and apparent or true solar time.

The two causes of the equation of time are—I. The obliquity of the ecliptic; II. the unequal motion of the sun.

There are thus three kinds of time—sidereal time, apparent solar time, and mean solar time.

The sidereal day is 23 hours 56 minutes, or 4 minutes shorter than the mean solar day of twenty-four hours. The difference arises from the sun's annual motion. If the sun and a fixed star were (if possible) observed to cross the meridian to-day simultaneously, to-morrow, after an interval of 23h. 56m., the fixed star will be on the meridian in the same position, but the sun will be slightly to the east of the meridian, and will not cross the meridian until four minutes after the star.

In 365 days these intervals of 4 minutes make up a day of 24 hours; consequently there is one more sidereal day in a year than there are solar days.

CHAPTER XV.—LATITUDE AND LONGITUDE.

To find the latitude at sea.

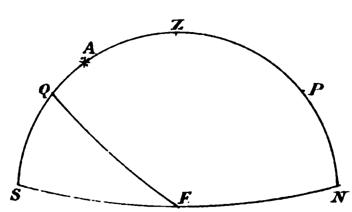

Fig LIX.

The simplest method is to observe the sun's meridian altitude. Let the sun be observed at A and let A S be measured. Then

knowing the day of the year, we can find from the Nautical Almanac, the sun's declination, A Q, for that day, ∴ subtracting A Q from A S we know Q S. But Q S = P Z for ZS = 90° and P Q = 90° ∴ taking away the common part Z Q we obtain Q S = P Z but P Z is the co-latitude and is therefore known, and the latitude is found at once.

To find the longitude.

Since in its apparent motion the sun passes once round the earth in every twenty-four hours, it must pass over 15° every hour. In other words, if the sun be on our meridian at this instant, in an hour's time it will be on the meridian of a place, 15° to the west of our position, that is noon at the second place is an hour later than it is with us. Hence when with us it is noon, at the western place it is only eleven o'clock. If the difference in longitude of any two places is known, we can easily find their difference in time by allowing an hour for every fifteen degrees difference in longitude. Conversely (a much more practically important matter) having given the difference in time for any two places, we can calculate this difference in longitude by allowing fifteen degrees for every hour. Moreover if we take some place on the earth as starting point and can obtain the time difference between this place and any other place on the earth, we can find the longitude of that other place with reference to the selected place. Greenwich is the place chosen by the majority of the great naval nations. Thus the problem of finding the longitude of any place resolves itself into the question: what is the difference between the local time of a particular place, and the corresponding Greenwich time? If the time difference be found a sum in proportion obtains the longitude. If local time be ahead of Greenwich time the place is to the east; if behind, to the west. Thus, Dublin time is about twenty-five minutes behind Greenwich time, *i.e.* when it is noon at Greenwich it is only 11.35 in Dublin.

The finding of the longitude, being reduced to the finding of the difference between local time and Greenwich time, there are three principal methods of solution.

I. By chronometers.
II. By lunar distances.
III. By eclipses of Jupiter's satellites.

I. To find the longitude by chronometer.

Ships generally carry one or more chronometers which are on starting set to Greenwich time. All that is necessary then is to obtain the local time, and compare it with the time indicated by the Greenwich chronometer; and the difference between the two times enables us to find the longitude.

To find the local time as required by this method, it is usual to observe the sun when on the meridian. If the instant of crossing the meridian can be found, it gives apparent solar noon; *correct by the equation of time for the particular day of the year,* according to the value given in the Nautical Almanac, and the result is the mean solar noon. This result should be then compared with the chronometers which indicate the mean solar time of the Greenwich meridian.

To find mean noon at sea. It is very difficult to obtain the exact instant at which the centre of the sun's disc crosses the meridian, by any of the methods of observation available at sea. The difficulty is solved as follows. Shortly before noon an altitude of the sun is taken (by Hadley's Sextant) and the time of the observation is recorded. Again shortly after noon, another observation is taken when the sun is at the same altitude as when first observed. Then the time of noon is half way between the two times of observation, for at equal intervals before and after noon the sun's altitudes are equal.

II. To find the longitude by lunar distances. Observe, by Hadley's Sextant, the distance of the moon from some bright star near it, and correct the observation for refraction and for parallax. Then in the Nautical Almanac will be found the Greenwich time at which the moon is at the particular distance thus found, from the fixed star. The local time of the observation compared with the Greenwich time as given by the Nautical Almanac determines the time difference and therefore the longitude.

To illustrate. The distance of the moon from Regulus when corrected for refraction and parallax is observed to be 10°, the time of the observation being quarter past two a.m. The Greenwich time as given in Nautical Almanac, at which the moon is 10° distant from Regulus is half-past twelve a.m. The difference between the Greenwich time and the local time is therefore an hour and three quarters. The corresponding difference in longitude is found by multiplying 15° by $1\frac{3}{4}$ and is therefore 26° 15′. Also since local time is ahead of Greenwich time the place is east of Greenwich. Finally the required longitude is 26° 15′ E.

III. To find the longitude by eclipses of Jupiter's satellites. The Greenwich times of the eclipses of Jupiter's satellites are given in astronomical tables. Therefore to find the longitude by this method note the local time of an eclipse of one of the satellites; the tables give the corresponding Greenwich time, and the difference between the two times enables us to calculate the longitude.

Objection to this method. It is very difficult to determine the precise moment of an eclipse, as it does not occur suddenly, but the satellite disappears gradually in Jupiter's shadow.

EXAMINATION QUESTIONS IN ASTRONOMY.

1. Explain by a figure the meaning of the terms—Right ascension, declination, latitude, longitude, azimuth, altitude.

2. Draw a diagram showing the relative positions of the equator, horizon, and meridian, for an observer within the arctic circle.

3. Draw a diagram showing the relative positions of the equator, horizon, meridian, and diurnal path of the sun on the day of the summer solstice, the observer being supposed on the tropic of Cancer.

4. What is the meaning of the terms—meridian, solstice, longitude, month, year? Note the ambiguities.

5. What is the meaning of the term obliquity of the ecliptic? What is its amount?

6. Explain accurately the terms—latitude, declination, azimuth, as applied to objects on the celestial sphere.

7. Illustrate by a diagram the celestial sphere as it appears to a spectator at Rio de Janeiro, latitude, 22° 54′ S.

8. What is the limit of north declination up to which stars of the northern hemisphere will be visible in latitude 22° 54′ S?

9. What is meant by the pole of the heavens?

10. Let a triangle be formed by joining the celestial pole, the zenith, and a star; write on the sides their value in terms of altitude, latitude, and declination: by what names are the angles called?

11. Draw a diagram exhibiting a star's right ascension, declination, latitude, and longitude; the spectator being supposed in latitude 60° N.

12. How has the figure of the earth been determined?

13. Assuming the earth to be a sphere, show that as an observer moves north or south of the equator on a meridian, the change in the altitude of the pole is equal to the change in his latitude. Also show how this fact enables us to determine the absolute size of the earth.

14. At what places on the earth can the ecliptic coincide with the horizon?

15. What is the reason that the earth is bulged at the equator?

16. Prove that the altitude of the pole is equal to the latitude.

17. How are the figure and magnitude of the earth ascertained?

18. Prove that if the earth be a sphere, the change in the altitude of the pole will be proportional to the space travelled over north or south on the same meridian.

19. How may the distance of the zenith from the pole be found from meridional observations of a circumpolar star?

20. Describe Eratosthene's method of finding the circumference of the earth?

21. Give the arguments for the earth's spherical shape, and show how its magnitude is found.

22. Prove the formula which connects the latitude of a place with the observed meridian altitude and the declination of a heavenly body.

23. How is it proved that the earth's diameter is about 8000 miles?

24. Show that the hour of sunrise on any particular day of the year may be calculated from a triangle on the sphere.

25. The sun rose this morning at 8.10 a. m. and sets at 3.38 p. m., determine at what hour he crosses the meridian.

26. The latitude and longitude of Bombay are 18° 56′ N. and 72° 53′ E.; find the meridian altitudes of the sun at the solstices.

27. What are the greatest and least meridian altitudes of the sun at a place whose latitude is 53° 25′?

28. The maximum meridian altitude of the sun at a certain place is 78° 28′; what is the latitude of that place?

29. What is the azimuth of the sun at sunrise at the vernal equinox?

30. Show by a diagram, and some notes on it that the shortness of this day, Dec. 9th, as compared with the length of to-night, follows from the theory of the sun's motion in the Ecliptic.

31. The meridian altitude of a star is observed at Turin, and proves to be 59° 18′. The declination of the star is known to be 14° 12′ S. Hence calculate the latitude of Turin. Is this calculation confirmed by finding that the meridian altitude of a star whose declination is 14° 12′ N. is 30° 54′?

32. By what observations is the declination of a star ascertained?

33. Find the sun's declination when he attains a meridian altitude of 80° at a latitude of 20° N.

34. Find the latitude at which the sun attains a meridian altitude of 20° when his declination is 18° N.

35. The meridian altitude of a star is observed at Dublin and is found to be 22° 45′. Its south declination is known to be 13° 55′. Calculate from these data the latitude of the observer.

36. Show that the effect of refraction is to raise the position of a star towards the zenith by an angle which varies as the tangent of the apparent zenith distance : and show how the coefficient of refraction is determined.

37. The coefficient of refraction of the atmosphere can be determined by observations of a circumpolar star. Show this, and describe Bradley's method of supplementing these observations, when the latitude of the place is not accurately known.

38. State and prove the law of refraction of the atmosphere.

39. If the sun's declination on any night be 10°, find the lowest latitude at which twilight will last all that night.

40. Assuming the law of atmospheric refraction $r = \kappa \tan z$, what is the numerical value of κ, and how ascertained ?

41. Find the duration of twilight at the equator at the time of the equinox.

42. Find the declination of the sun when twilight lasts all night at the latitude 53° 20′.

43. In what way may the coefficient of refraction be determined when the latitude of the observatory is known ?

44. Show that twilight will begin to last all night when the sum of the latitude of the place and the declination of the sun is equal to 72°.

45. Determine the sun's declination when twilight begins to last all night at Dublin (lat. 53° 23′).

46. Prove that the atmospheric refraction varies as the tangent of the zenith distance.

47. State the cause of twilight. When does it begin and end, and why is its duration much less in the tropics than in the higher latitudes ?

48. By what observations on the moon and a neighbouring fixed star is the parallax of the former body measured ?

49. What is annual parallax, and how is it found ?

50. What effect does diurnal parallax produce upon the apparent position of the moon, and to what is it proportional ?

51. Find the distance of Sirius if the annual parallax be ·23″.

52. Assuming the radius of the earth to be 4,000 miles, and the moon's horizontal parallax to be 57′ 6″, calculate the moon's distance from the earth.

53. Define diurnal parallax, and calculate its amount in terms of the zenith distance.

54. Distinguish diurnal from annual parallax.

55. What is horizontal parallax ?

56. From two successive observations of Jupiter in quadrature, the distance of that planet from the sun may be calculated, the earth's distance from the sun being known.

57. Assuming the earth's diameter to be 8,000 miles, the horizontal parallax of Mars in opposition to be 15″, and the earth's distance from the sun to be 93,000,000 miles, calculate the distance of Mars from the sun.

58. Give Bessel's method for determining the annual parallax of a star.

59. By what observations have the distances of some of the fixed stars been determined ?

60. Show how the distance of the moon is to be ascertained.

61. How is the magnitude of the moon ascertained ?

62. If the annual parallax of a star be ·3″, find its distance.

63. Compare parallax and refraction as regards (*a*) the direction, (*b*) the causes, and (*c*) the law of variation, of the effects they produce.

64. A drop of rain falling vertically through a hole in the roof of a railway carriage strikes a passenger in the face : (*a*) What false conclusion does the passenger draw spontaneously with regard to the direction of the rain ? (*b*) Apply this illustration to the phenomenon called aberration.

65. Assuming that light takes 8m. 18s. to travel from the sun to the earth, prove the formula—

$$\text{Aberration} = 20''\!\cdot\!45 \sin(\text{earth's way.})$$

66. Describe the observations from which we calculate the size and distance of a planet such as Mars, when we know the size of the earth. How would the observations differ if the planet were Jupiter ?

67. State the argument for the annual motion of the earth derived from the aberration of the fixed stars.

68. What are the experimental proofs of the earth's diurnal motion ?

69. Explain the phenomenon of the seasons.

70. State some of the proofs of the earth's rotation.

71. What is the right ascension of a star, and how is it determined ?

72. How is it proved that the sun rotates ?

73. Describe the course of the seasons as observed at a place on the Arctic circle.

74. Describe the phenomena called precession and nutation, and explain whence they arise.

75. How has precession affected the first point of Aries ?

76. Show how the change of seasons depends on the inclination of the earth's axis to the orbit round the sun.

77. The star α Lyræ will be within 5° of the North Pole in 10,000 years ; to what is this to be attributed ?

78. State accurately the phenomenon known as precession of the equinoxes and state its effects on the latitude and longitude of a fixed star.

79. Assuming Kepler's Third Law, connecting the periodic time of two planets with their mean distances from the sun, prove that their velocities are inversely proportional to the square roots of their distances.

80. Prove that the illuminated portion of a planet varies as the external angle of elongation at the planet between the sun and earth.

81. Show how to find the ratio of the distances of a superior planet and the earth from the sun.

82. If v, v′ be the velocities of two planets, and r, r′ their distances from the sun, prove that $v^2 : v'^2 :: r : r'$.

83. State Kepler's second law, and explain by its aid why a planet is moving most rapidly when nearest to the sun.

84. Show how the sidereal period of a planet may be determined from the synodic period ; and prove that the sidereal period of Venus is 224·7 days, its synodic period being 584 days.

85. If the interval between two inferior conjunctions of Mercury be 115·877 days, determine its periodic time.

86. Show that the periodic time (P) of an inferior planet is given by the equation.

$$P = \frac{E\,T}{E+T}$$

where E = periodic time of the earth, and T = interval between two successive conjunctions.

87. The periodic time of Uranus is 82 of our years. By what observations and calculation is this proved ?

88. In what way may the periodic time of a superior planet be found ?

89. State as nearly as you can the distances of the following bodies from the sun,—(*a*) the earth, (*b*) Neptune, (*c*) any fixed star.

90. Why is the planet Mercury so seldom seen ?

91. What are Kepler's laws of planetary motion ?

92. How may the ratio of the distances of Venus and the Earth from the Sun be found by observation and calculation ?

93. The mean distances of the Earth and Mars being in the ratio of 10 : 15.2 determine the periodic time of Mars in days.

94. Find, by Kepler's third law, the distance of Neptune, if his periodic time be assumed to be 165 years.

95. The Earth if seen from Mars would appear to be twice as large as Mars

appears to be as seen from the Earth : from this fact determine the real diameter of Mars.

96. Explain the phases of the Moon.

97. If the moon's sidereal period be 27 d. 7 h. find the interval between two full moons.

98. If the sun and moon are visible at the same time, how would you know whether the moon was in ascending or descending phase ?

99. What is the phenomenon called harvest moon ?

100. Show that the moon has no appreciable atmosphere.

101. Explain by a diagram the moon's libration in latitude.

102. State how the eclipses of Jupiter's satellites enabled Roemer to discover the velocity of light.

103. Explain how the velocity of light affects the position of the fixed stars.

104. How may the velocity of light be calculated from the aberration of the fixed stars.

105. On what circumstances does it depend whether the annual aberration of a star will be circular, elliptic, or linear ?

106. How does the velocity of light account for the aberration of the fixed stars ?

107. How may the velocity of the earth be calculated when we know the velocity of light and the amount of aberration of a star ?

108. Calculate roughly the earth's velocity, in miles per second, through space, and hence determine the coefficient of aberration.

109. Show how the occurence of a lunar eclipse depends on the magnitude of the moon's latitude at opposition, as compared with those of the lunar and solar parallaxes, and the apparent semi-diameters of the sun and moon.

110. Describe the circumstances under which an eclipse of the moon takes place, and explain why there is not a lunar eclipse at every full moon, and a solar eclipse at every new moon.

111. Show that an annular eclipse of the moon is impossible.

112. Explain what is meant by the lunar ecliptic limits, and show how they may be found.

113. Why is an eclipse of the sun sometimes total, and sometimes only annular ?

114. Show how the length of the moon's shadow, when in conjunction, may be found.

115. What are the lunar ecliptic limits, and how found ?

116. The major and minor lunar ecliptic limits are respectively about $11\frac{1}{2}$ and $9\frac{1}{2}$ degrees. What is the meaning of the statement ? How is its truth proved ?

117. How may the angle which the section of the earth's shadow at the distance of the moon subtends at the earth be found? from knowing the parallaxes of the sun and moon, and the sun's apparent diameter.

118. Determine the two angular breadths of the cone of common tangents to the earth and sun at the distance of the moon in terms of the two parallaxes and the sun's semi-diameter.

119. What is the cause of a solar eclipse?

120. If the moon's parallax be 1° 0′ 13″, the sun's parallax 8″, 86, and the sun's semi-diameter 15′ 45″ calculate the angle which the semi-diameter of the section of the earth's shadow at the moon's distance, subtends at the centre of the earth.

121. What are the two causes of the equation of time?

122. The mean length of a solar day exceeds that of the sidereal day by 3′ 56″ nearly. Hence, show that any quantity of sidereal time is converted into mean time by multiplying the former by a fraction very nearly equal to 365 : 366.

123. What is meant by the equation of time, and on what two causes does it depend?

124. Explain clearly the two causes which make time, as observed by a sun dial, differ from that given by a clock.

125. Explain how observations of the transit of Venus enable us to calculate the sun's distance?

126. Describe the transit of Venus, and show how, by its mean, the distance from the earth to the sun is determined.

127. Find the sun's distance if his parallax be 8″·76.

128. Explain how the sun's distance may be found from observations on Mars when nearest to the earth.

129. Explain the method by which Delisle proposed to find the sun's distance.

130. Explain any method of finding our distance from the sun.

131. What is the line of collimation of a telescope?

132. In what respects is it more advantageous that a telescope should be mounted on a circle than on a quadrant?

133. Explain the instrument by which the declinations of the stars may be found.

134. What is meant by the right ascension and declination of a star? By what astronomical instruments are they detetermined, and in what manner?

135. Describe Hadley's sextant.

136. State some of the corrections to be made, or precautions to be taken, in using a transit instrument.

137. For what purpose is the sextant used?

138. How is the line of collimation of a telescope placed in the meridian?

139. How would you ascertain your latitude at sea?

140. How is the latitude found from the meridian altitudes of a circumpolar star?

141. Describe any method of finding the longitude of a vessel when at sea.

142. Explain how the latitude can be measured in an observatory.

143. What o'clock is it at New York (74° 8′ W.) when it is 2 P.M. at St. Petersburg (30° 19′ E.)?

144. What o'clock is it at St. Petersburg (30° 19′ E.) when it is 2 A.M. at New York (74° 8′ W.)?

145. Explain the method of finding longitude at sea by observations of Jupiter's satellites.

146. Find what o'clock it is at Bombay (72° 53′ E.) when it is 2h. 30min. A.M. at Dublin, whose longitude is 6° 20′ W.?

147. Knowing the Plough, where would you look on a bright night for the stars Capella, Vega, Arcturus, Aldebaran, Sirius Major, and the constellations Casseopeia, Pegasus, and Hercules?

148. Draw a sketch of the seven principal stars in the Great Bear, and show the "Pointers" and the Pole Star.

149. What is remarkable about the star called *Mira*?

150. What is meant by binary stars?

151. What are periodical stars?

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